

# Marine Biogeochemical Modeling: Basic Philosophy and Methods

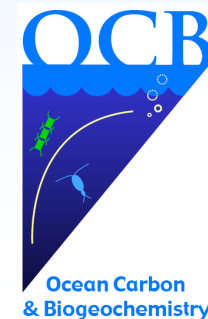
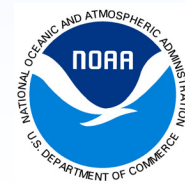
**Scott Doney (WHOI)**

OCB Ocean Acidification Short Course 2009

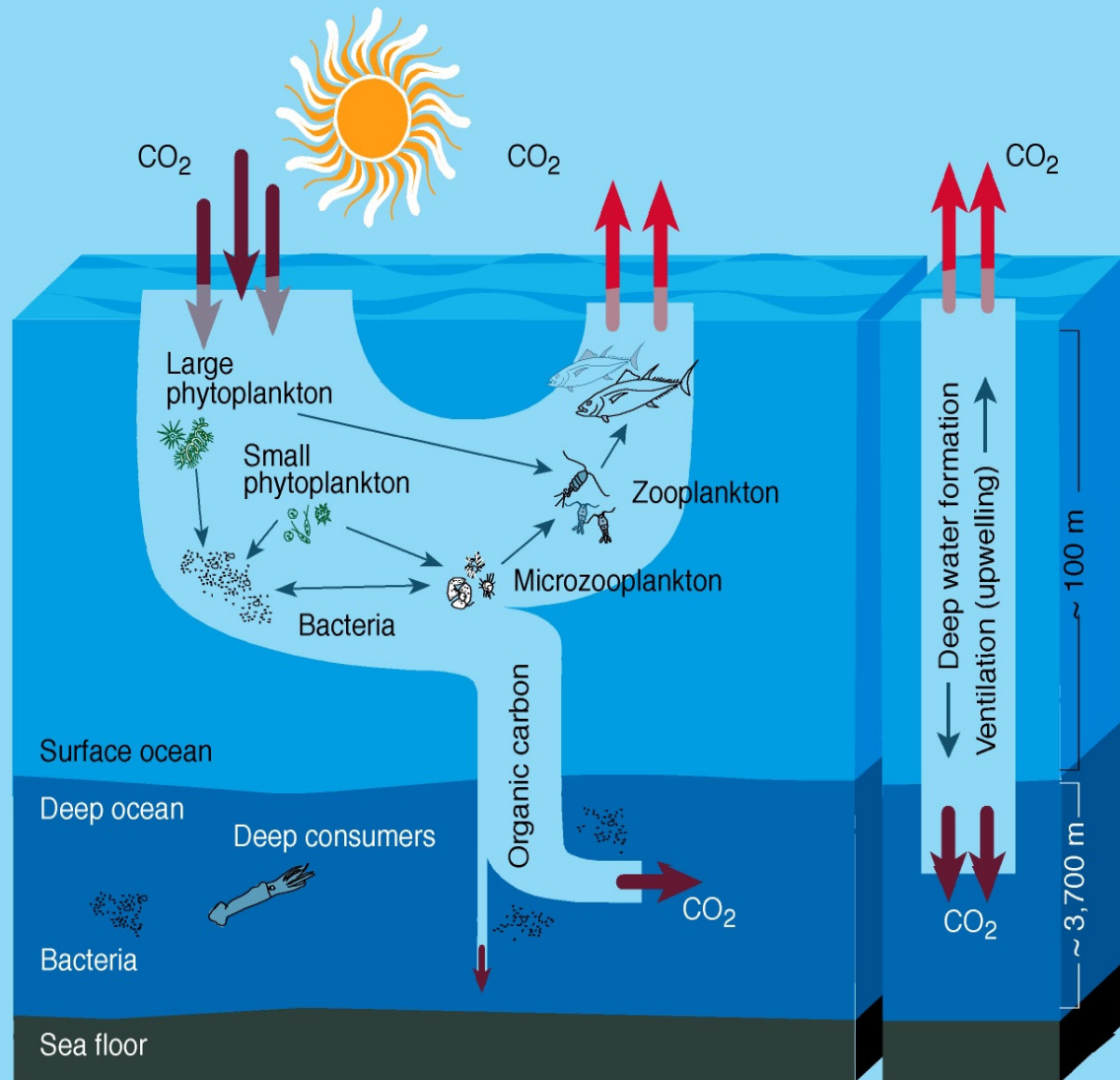
## Talk Outline

- Science Motivation
- Philosophy of Modeling
- Model Construction
- Evaluation Against Data

Supported by:



# Ocean Carbon Cycle & Ocean Acidification



Anthropogenic CO<sub>2</sub> uptake currently controlled by ocean circulation; but in future, what will be role of climate & biology?

For ocean acidification may want models to address many different aspects:

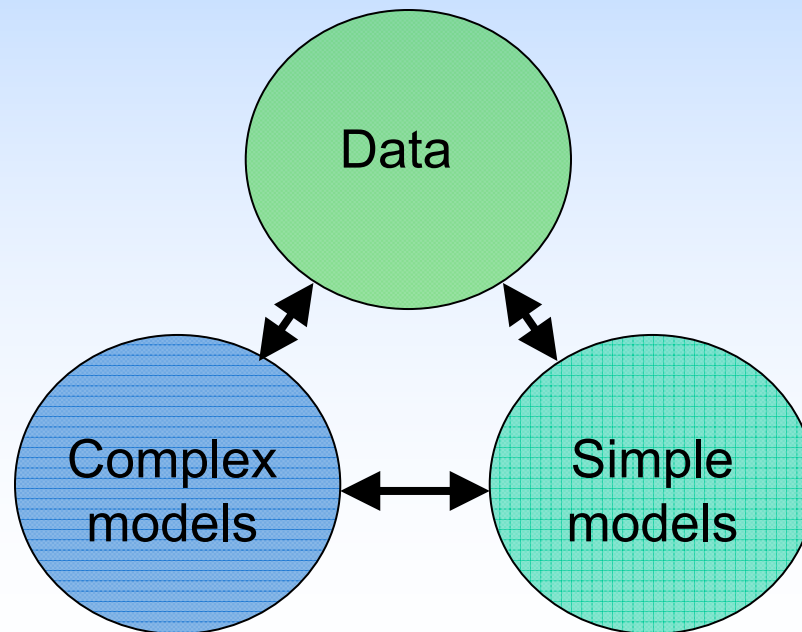
- patterns & trends in seawater chemistry*
- population biology of individual species*
- food-web & ecological interactions*
- biogeochemical feedbacks*
- socio-economic effects on fisheries & ecosystem services*

"I am never content until I have constructed a mechanical model of the subject I am studying. If I succeed in making one, I understand; otherwise I do not."

- Lord Kelvin

"People don't understand the earth, but they want to, so they build a model, and then they have two things they don't understand,"

-Gerard Roe in "The Whale and the Supercomputer" by C. Wohlforth



# The Interdisciplinary Conundrum

## Physics

- (relatively) simple set of (mostly) known governing equations (Navier-Stokes) => complex phenomenon
- unresolved scales

## Chemistry

- mass balance equations
- chemical fields integrate over time/space variability
- don't uniquely identify process/mechanism

## Biology

- information as stories and conceptual pictures
- historical/evolutionary contingency
- no “biological” Navier-Stokes
- aggregate complicated dynamics across multiple-scales (genes, cells, populations, ecosystems)

# Why do we build/use models?

## -Quantitative dynamical framework

- Are different data sets, rate estimates consistent?

## -Design of experiments & observing systems

- What, where and when do we sample?

## -Hypothesis testing

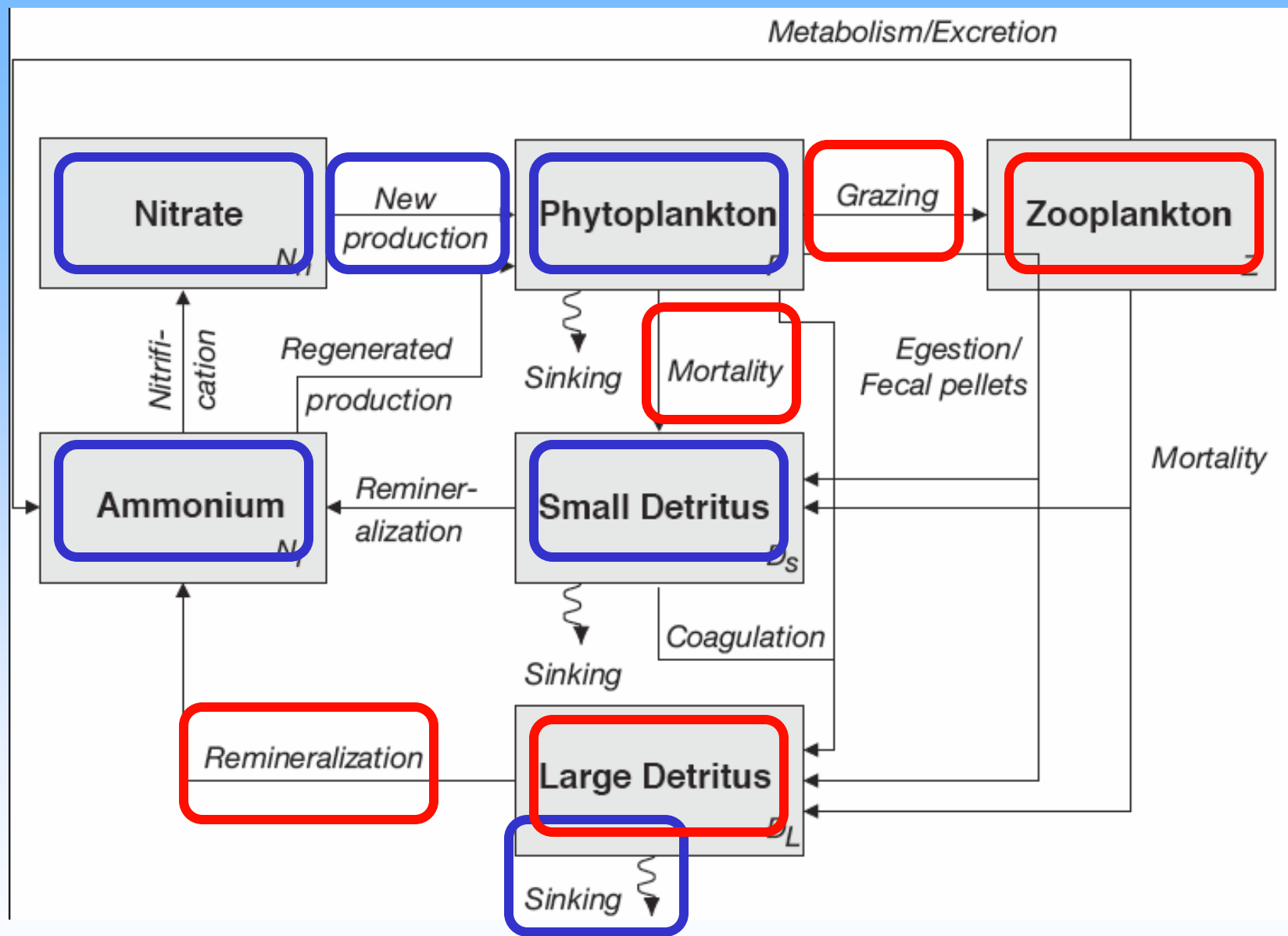
- If we add or change X, what happens?

## -Forecasting

- What will the ocean look like at some point in the future?

## -Solving for unknown parameters and rates

- Given things we can measure, can we estimate  
can we estimate properties that are difficult to  
measure?



“Stocks” versus “Rates”  
 C  $dC/dt$

Estimated from data  
 Unknowns

# What is a model?

## -regression curves

variable Y is a function of other variables X and regression parameters p:

$$Y=f(\text{variables, parameters})$$

$$\text{Chl} = p_0 + p_1 T$$

## -forward models (*often time dependent*)

Integrate forward in time to find Y:

$$dY/dt = \text{circulation} + f(\text{variables, parameters, forcing})$$

$$d \text{ Phyto } /dt = \text{circulation} + \mu * \text{Phyto} * (1 - e^{-\alpha E / \mu}) - \lambda \text{Phyto}$$

## -inverse models

Invert the problem to find parameters from data:

$$\text{Parameters} = f(\text{circulation, data, forcing})$$

# What is a model? (continued)

## -diagnostic versus prognostic models

In diagnostic models, some variables may be prescribed based on observations:

e.g., satellite chlorophyll => ecosystem model  
(no equation for  $d\text{Chl} / dt$ )

## -data assimilation

Combine model equations and observations in a dynamically consistent fashion

e.g., weather prediction  
analysis =  $f(\text{model forecast, observations})$

Prognostic, forward models needed to project into the future

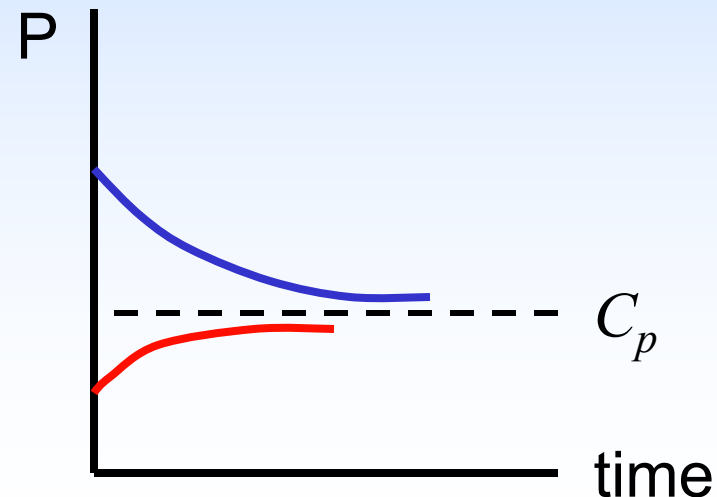
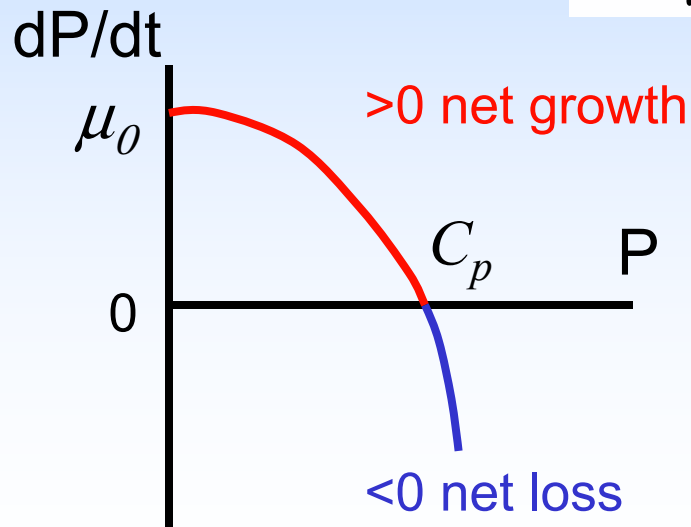


# From Word Problem to Equations

- Phytoplankton levels depends on nutrient inputs
- Perturbations relax back to some stable background level

$$\frac{dP}{dt} = \mu_0 \left( 1 - \frac{P}{C_p} \right) P$$

- **Functional form**  
Logistic model
- **State variable** (concentrations)  
 $P$  (mmol C/m<sup>3</sup>) phytoplankton
- **Parameters**  
 $\mu_0$  (1/d) and  $C_p$  (mmol C/m<sup>3</sup>)



# Simple NPZ Model

$$\frac{dP}{dt} = \mu_0 \left( \frac{N}{k_N + N} \right) \left( 1 - e^{\alpha E / \mu_0} \right) P - g \left( \frac{P}{k_P + P} \right) Z - m_P P$$

Nutrient limitation      Light limitation      Grazing      Mortality

$$\frac{dZ}{dt} = ag \left( \frac{P}{k_P + P} \right) Z - m_Z Z$$

$$\frac{dN}{dt} = -\mu_0 \left( \frac{N}{k_N + N} \right) \left( 1 - e^{\alpha E / \mu_0} \right) P + (1 - a) g \left( \frac{P}{k_P + P} \right) Z + m_P P + m_Z Z$$

- Three coupled ordinary differential equations
- Mass conservation

# Discretization & Numerical Integration

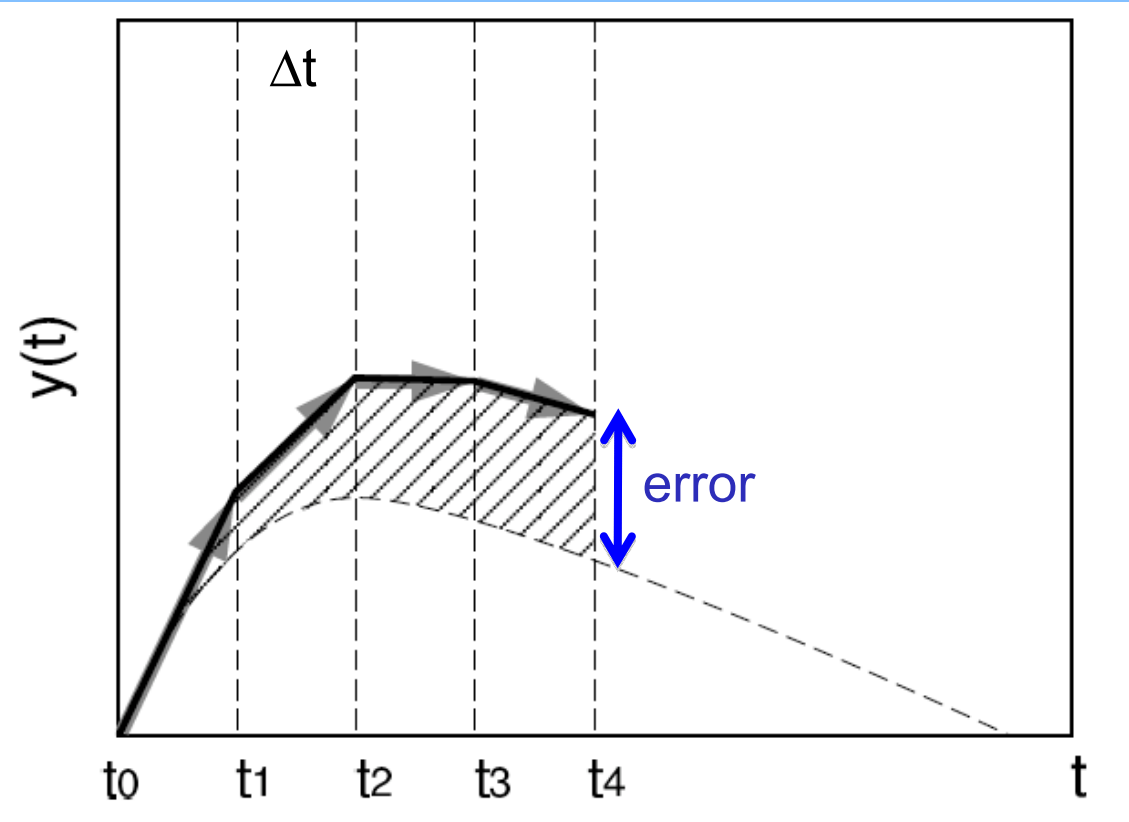
$$\frac{dy}{dt} = f(t, y)$$

Discrete form

$$\Delta y = f(t, y) \Delta t$$

numerical method:  
Euler's Method

$$y^{n+1} = y^n + f(t^n, y^n) \Delta t$$



- Subdivide time into discrete time-steps
- “Integrate” forward using  $\Delta y/\Delta t$  approximation
- Numerical methods introduce errors

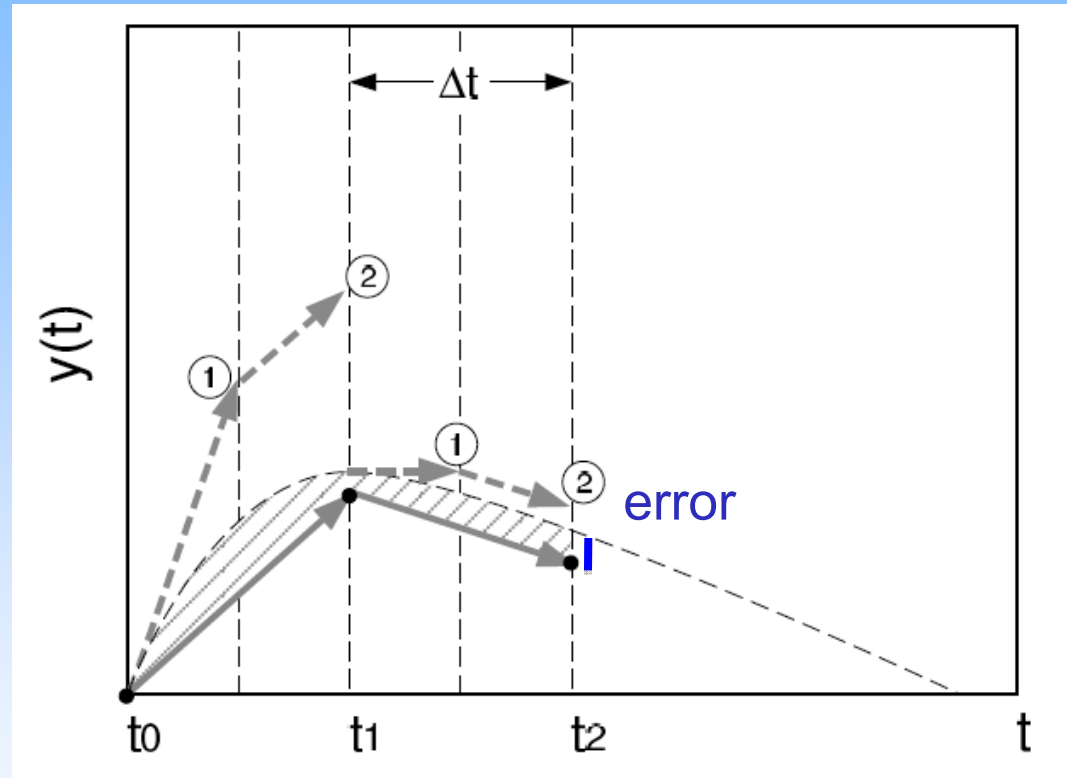
# Discretization & Numerical Integration

Runge-Kutta (2<sup>nd</sup> order)

$$k_1 = f(t^n, y^n)\Delta t$$

$$k_2 = f\left(t^n + \frac{1}{2}\Delta t, y^n + \frac{1}{2}k_1\right)\Delta t$$

$$y^{n+1} = y^n + k_2$$



- use a “trial step” to find gradient at mid-point
- even with 2x larger  $\Delta t$ , more accurate integration

# How do you estimate parameters and functional forms?

## Laboratory & field incubations

- P-E curves; nutrient uptake curves
- elemental stoichiometry

## Comparative analysis

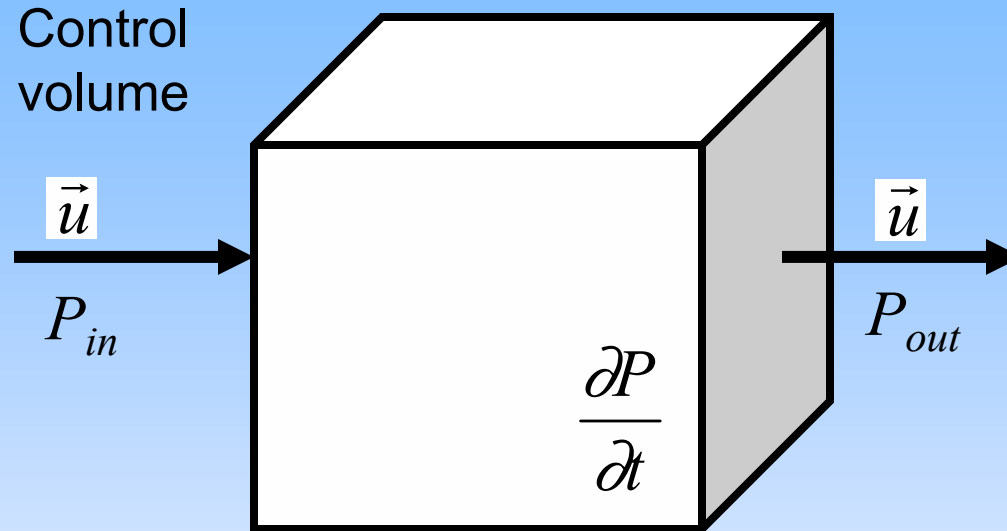
- allometric relationships

## Tuned or optimized against field data

- mismatch between parameters and data
- cross-site comparison

## Previous models

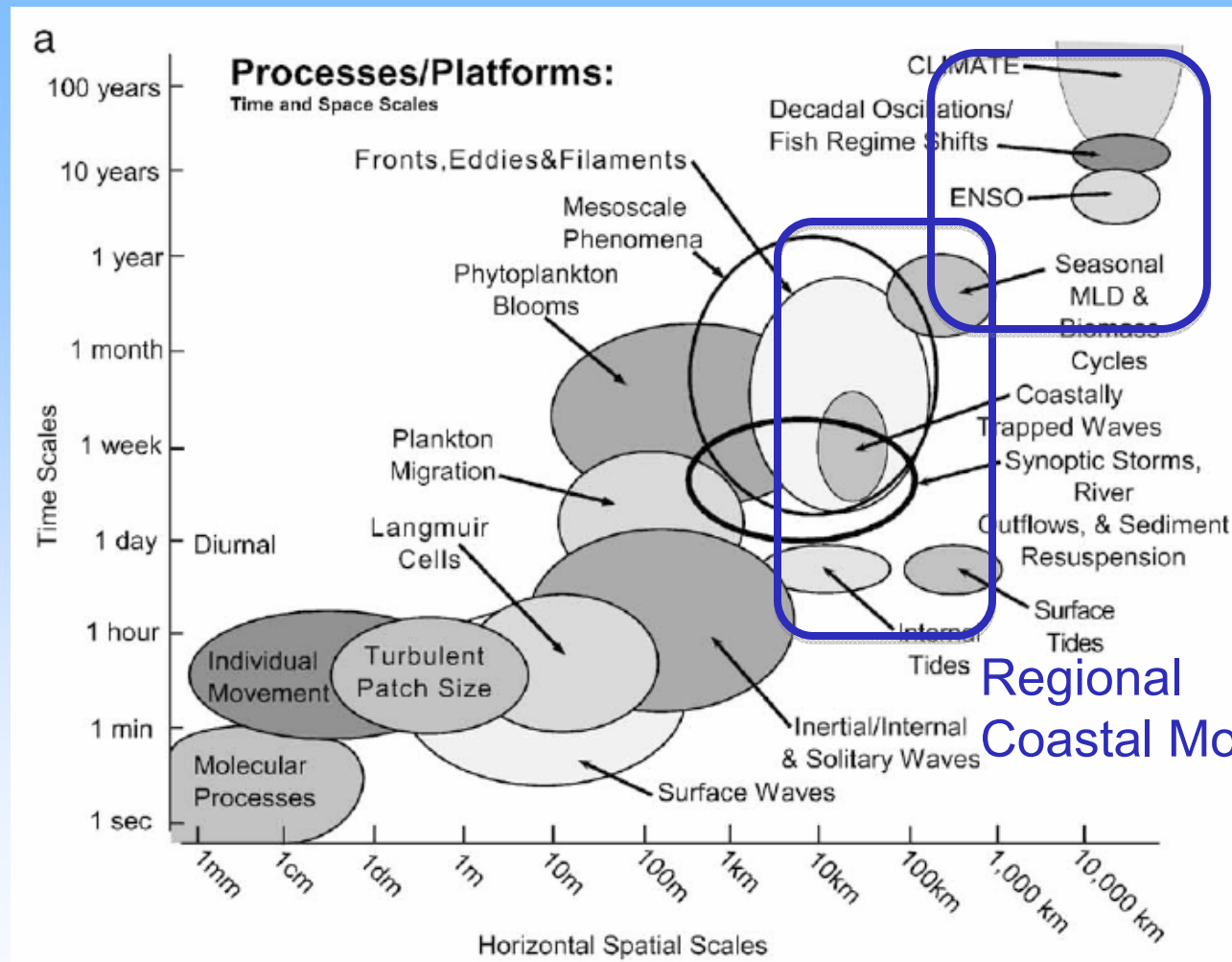
# Adding Circulation



$$\frac{\partial P}{\partial t} + \vec{u} \cdot \nabla P - \kappa \nabla^2 P = RHS$$

advection   diffusion   biological  
source/sink  
terms

# Models have time/space scale limits



Global  
Climate  
Model

Regional  
Coastal Model

Dickey  
(2003)

- Computational costs scale as  $(\text{length})^3$  to  $(\text{length})^4$  and (time)
- Typically get ~2-3 decades in space (more in time)
- Can not resolve all scales; parameterize sub-grid scale

# Coupled “Eco-biogeochemical” Elements

## Physics (flow field; mixing)

- equations for resolved flow; level of approximation (e.g., primitive equations; quasi-geostrophy)
- forcing (winds, heat & freshwater fluxes, light, tides)
- parameterization of unresolved scales (mixing)
- model architecture (e.g., horizontal vs. isopycnal)

## Chemistry ( $CO_2$ , $O_2$ , nutrient fields)

- air-sea gas exchange
- elemental stoichiometries
- trace metal deposition and scavenging

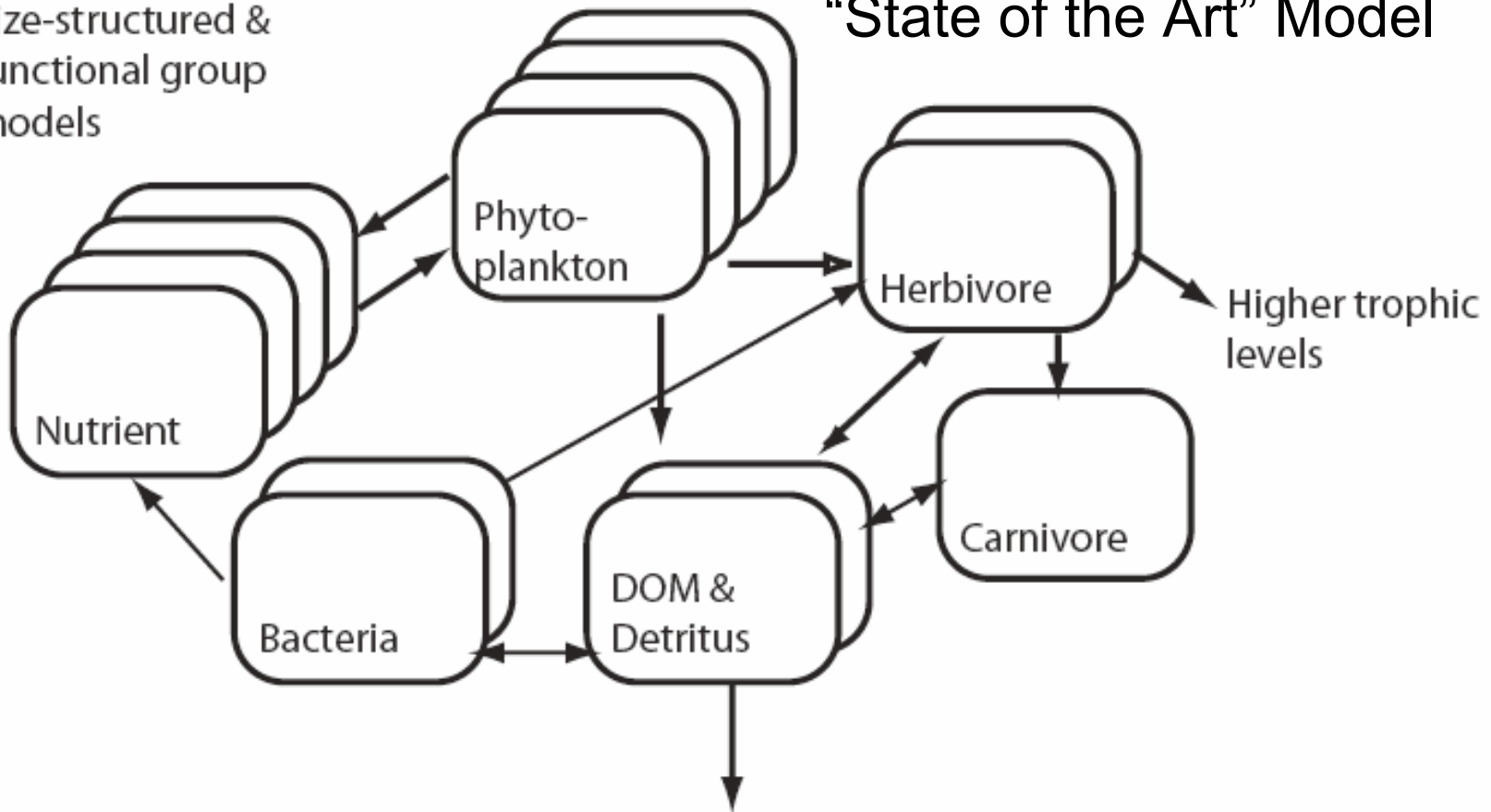
## Biology

- primary production, respiration, remineralization
- community structure and succession
- bio-optics
- etc.



Size-structured & functional group models

## “State of the Art” Model



- Aggregate into trophic levels/functional groups
- Rates/processes from limited culture/field studies
- Many aspects empirically based
- Data poor for validation (rates, grazing, loss terms)

# Multi-scale models

## -cellular models

- gene expression, metabolism, energetics

## -population models

- individual based or continuous dist.
- cell-cell interactions (LES & DNS)

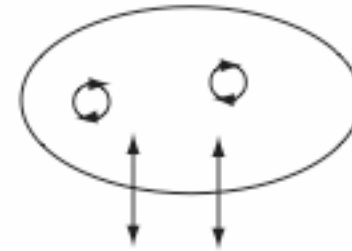
## -simulate ecological functions

- “genotype” => “phenotype”
- abandon “boxes”

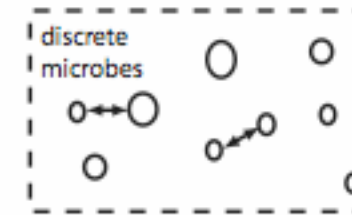
## -ecological/evolutionary rules for ecosystem assembly

- maximize resilience or energy flow
- emergent behavior & selection
- selection and niche adaptation
- Physiological plasticity & constraints
- micro/macroevoolution

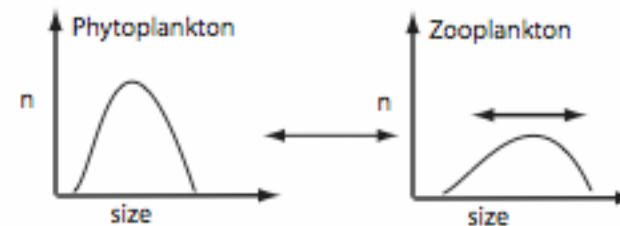
Cellular genomic, metabolic & energetic models



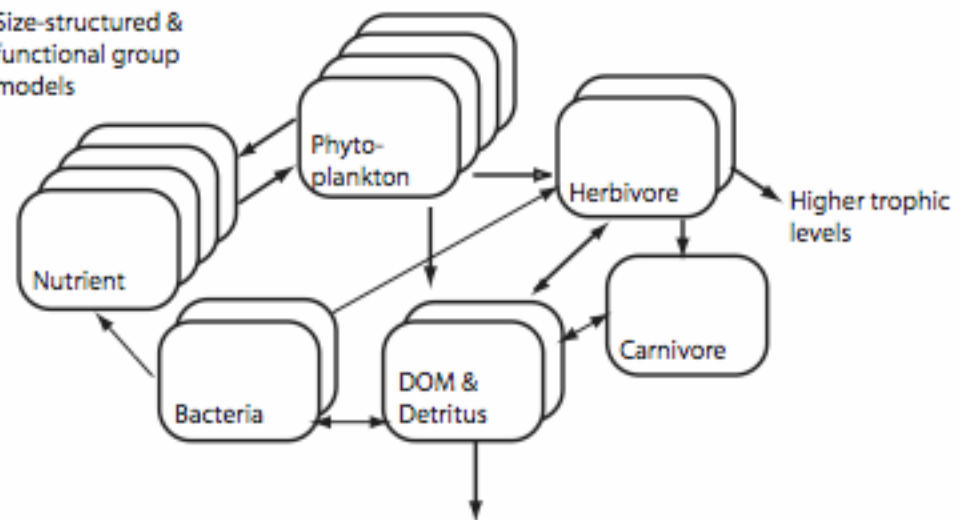
Individual-based & multi-scale models



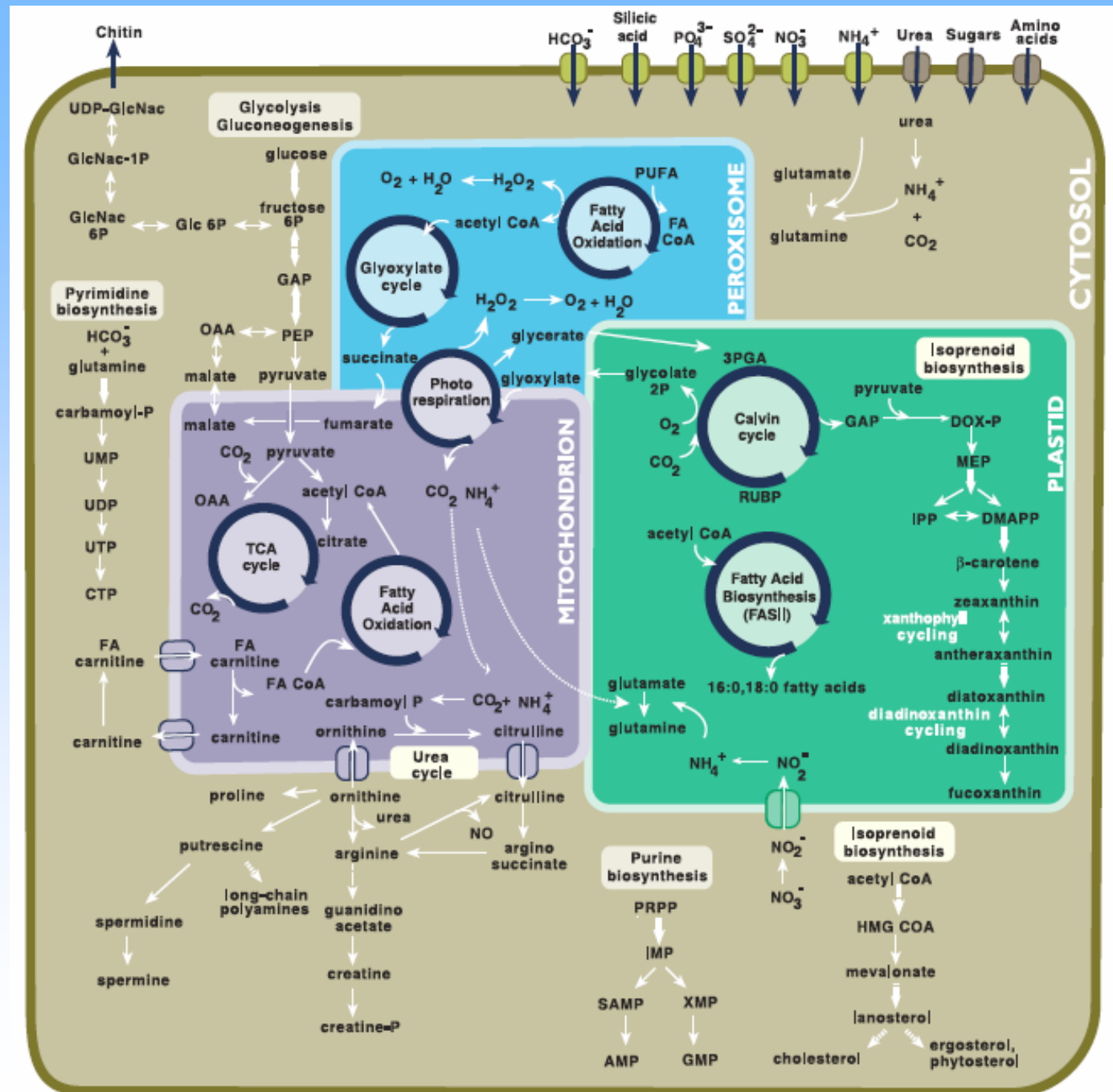
Continuous distribution models



Size-structured & functional group models

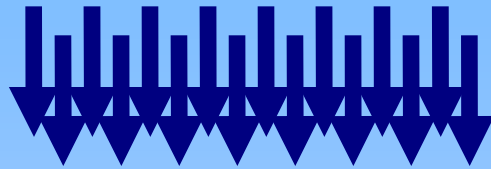


# Cell Physiology/ Genomics



*Diatom Genome*  
Armbrust et al.,  
Science, 2004

# Competition Based Approach



Genetics and  
physiology

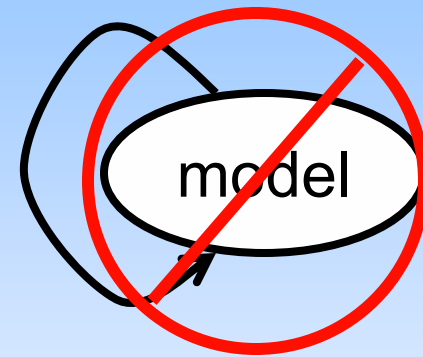
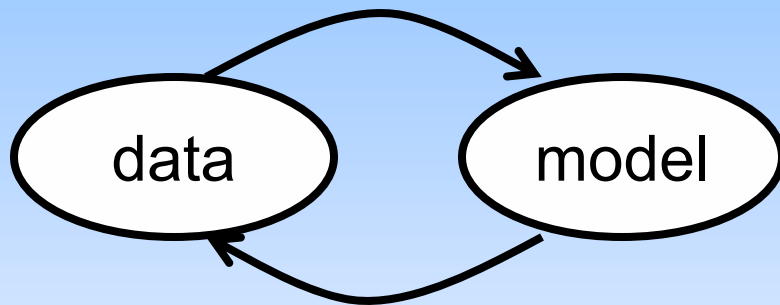
Environment

- Initialize many potentially viable functional types of phytoplankton
- Assign attributes and parameter values from prescribed ranges with element of chance
- ***Explicit competition selects for fittest functional types***

Competition  
Predation  
selection

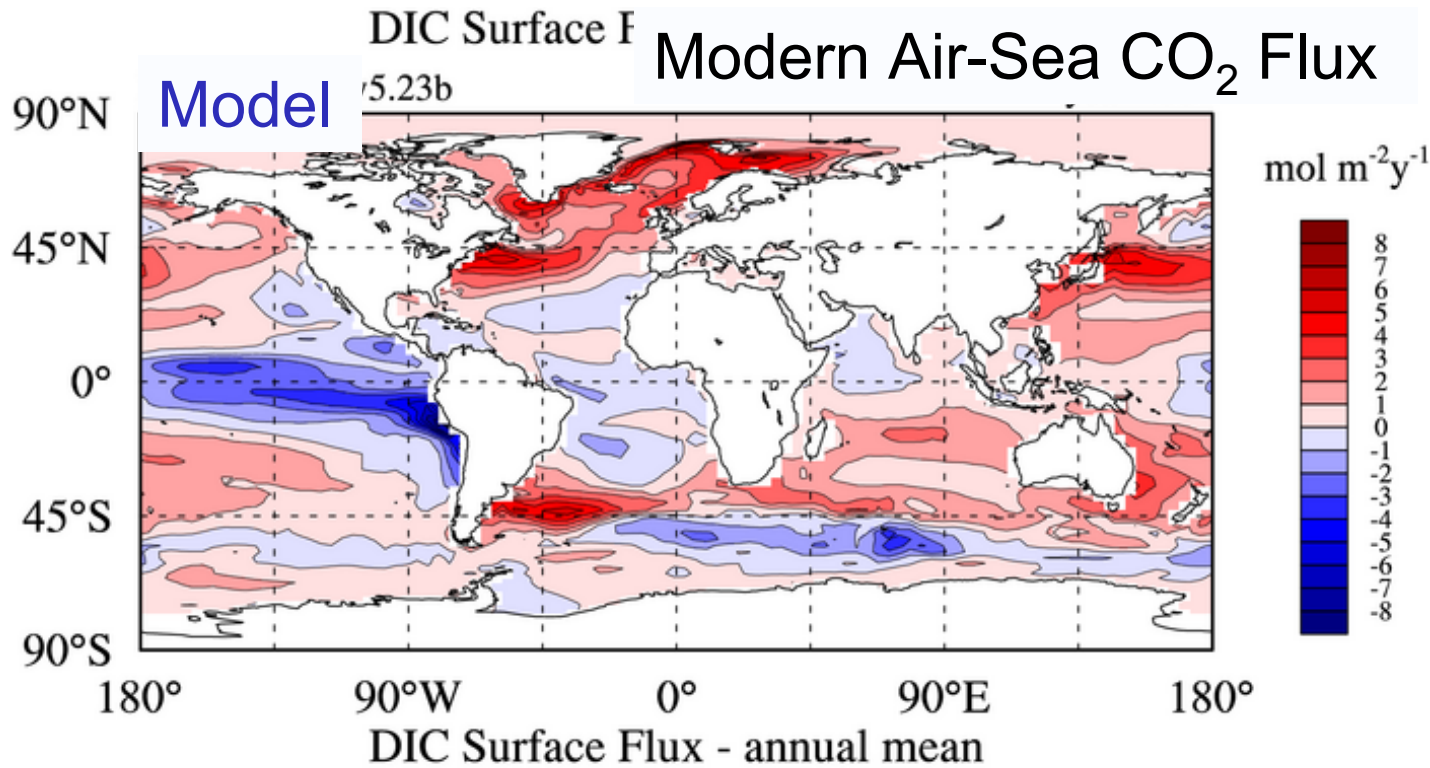
Ecosystem structure  
and function

# Ocean ecology and biogeochemistry are (still) data-driven sciences



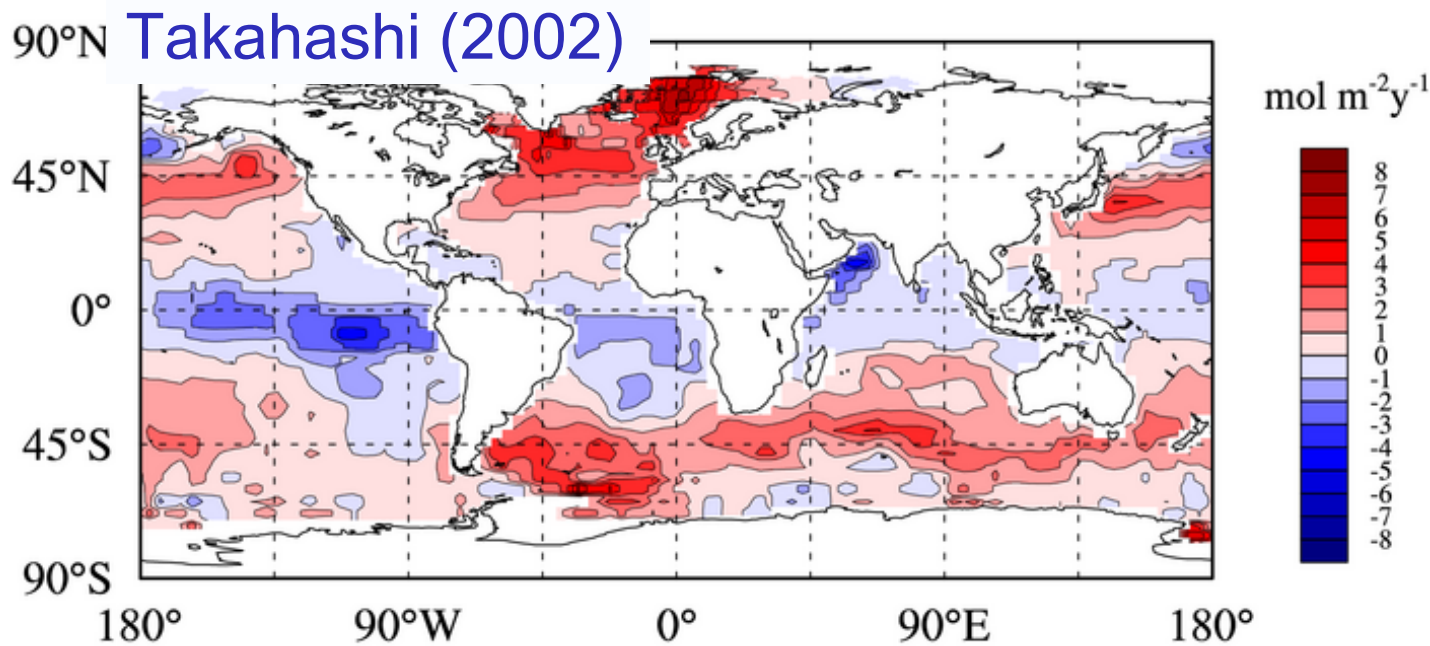
How do we avoid the trap of:  
“false models tested by inadequate data”

John Steele



“Looks pretty good” test

$\chi$  (chi) by eye



Doney et al. Deep-Sea Res. II 2009

Doney et al. J. Mar. Systems 2009

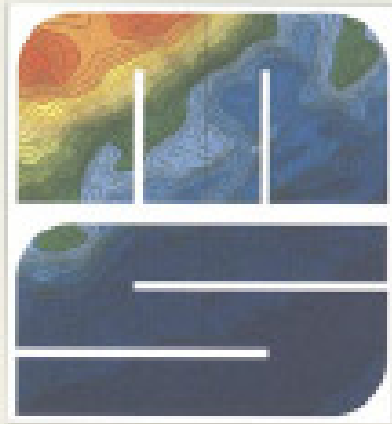




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Special Issue

Skill Assessment for Coupled Biological/Physical Models of Marine Systems

edited by

Daniel R. Lynch, Dennis J. McMillin, Jr. and Francisco E. Miller

# JOURNAL OF MARINE SYSTEMS

J. Marine Systems Special Issue on  
*Skill Assessment for Coupled Biological /  
Physical Models of Marine Systems*

Vol. 76, Issue 1-2, 2009



4) AE – the average error (bias)

$$AE = \frac{\sum_{i=1}^n (P_i - O_i)}{n} = \bar{P} - \bar{O},$$

Bias

1)  $r$  – the correlation coefficient of the model predictions and observations:

$$r = \frac{\sum_{i=1}^n (O_i - \bar{O})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^n (O_i - \bar{O})^2 \sum_{i=1}^n (P_i - \bar{P})^2}},$$

Correlation

2) RMSE – the root mean squared error (also referred to as root mean squared difference):

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (P_i - O_i)^2}{n}},$$

rms Error

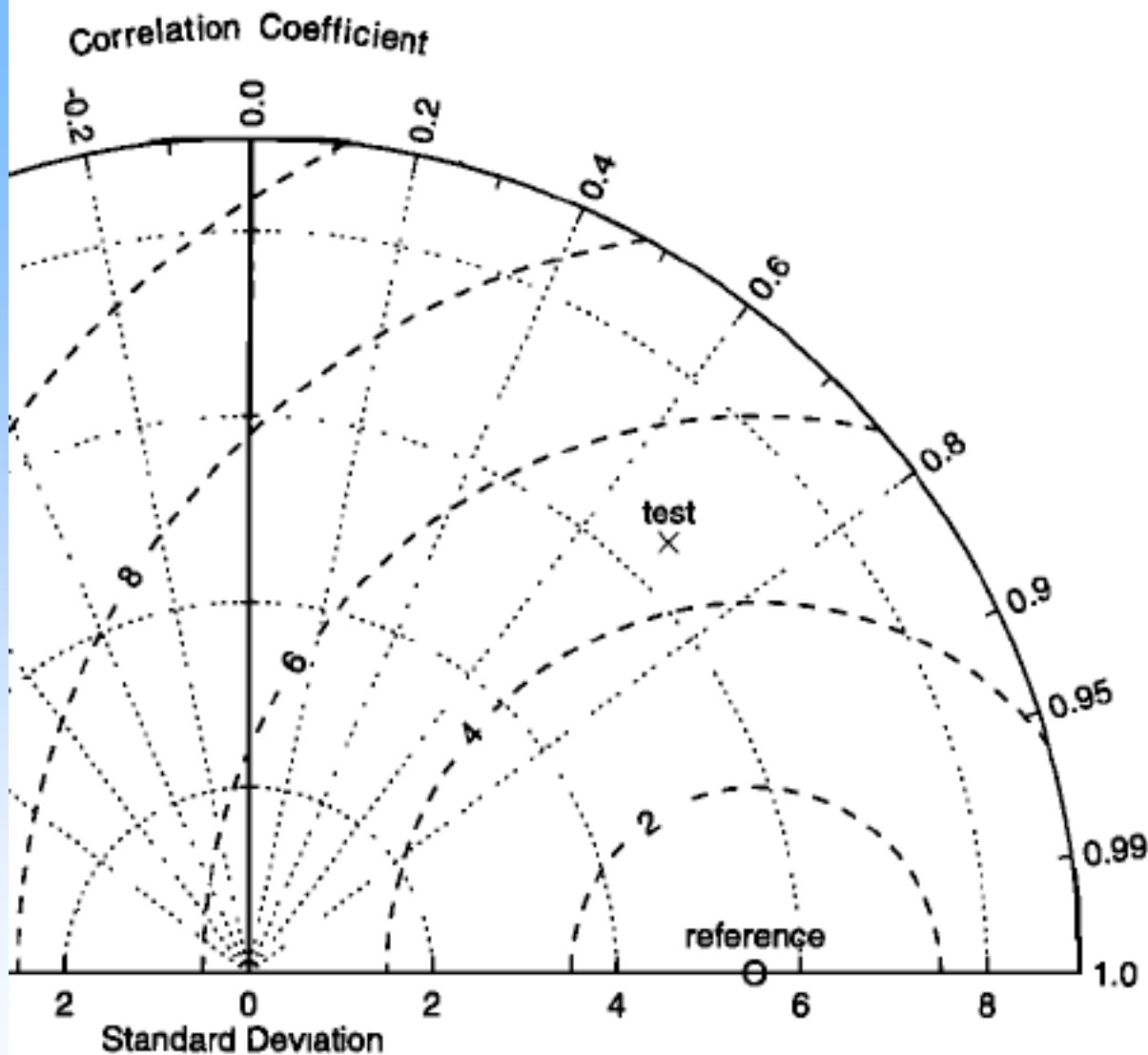
Doney et al. J. Mar. Systems 2009

Stow et al. J. Mar. Systems 2009



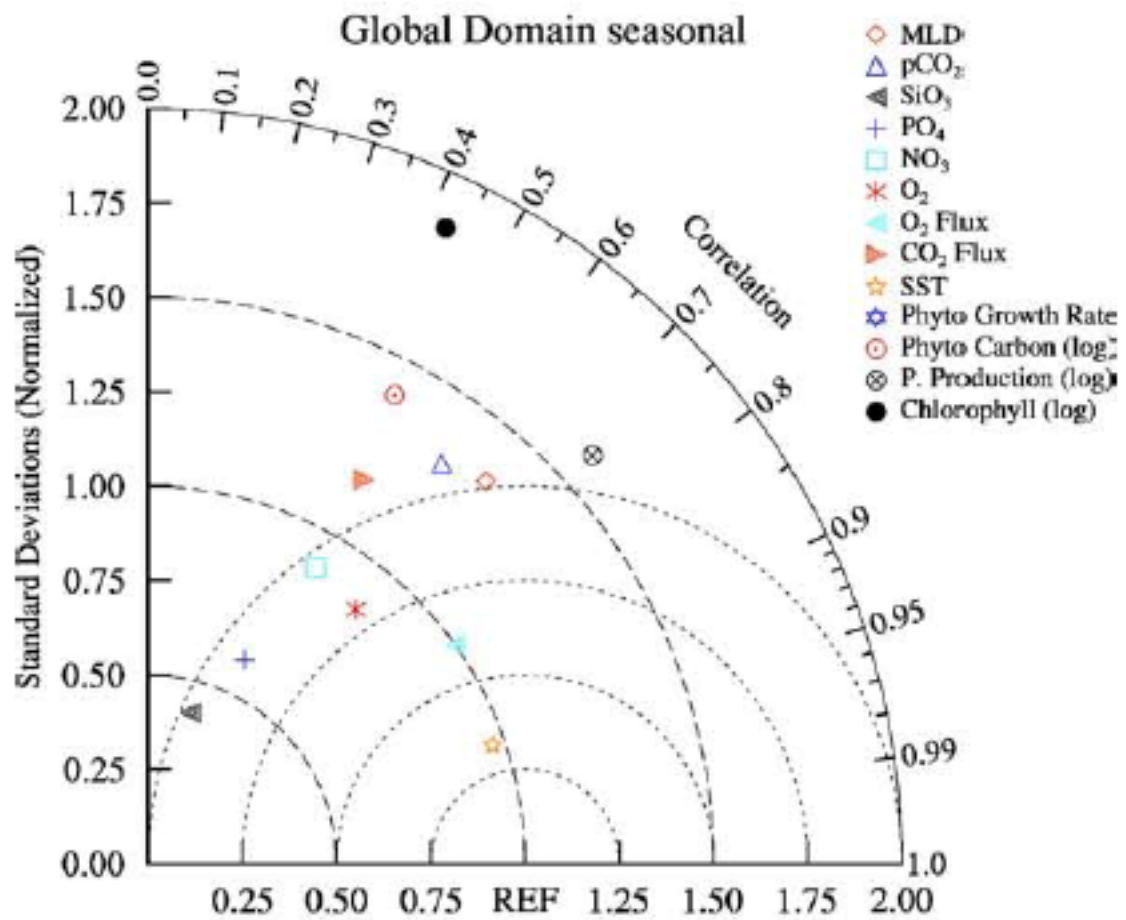


# Taylor Diagram

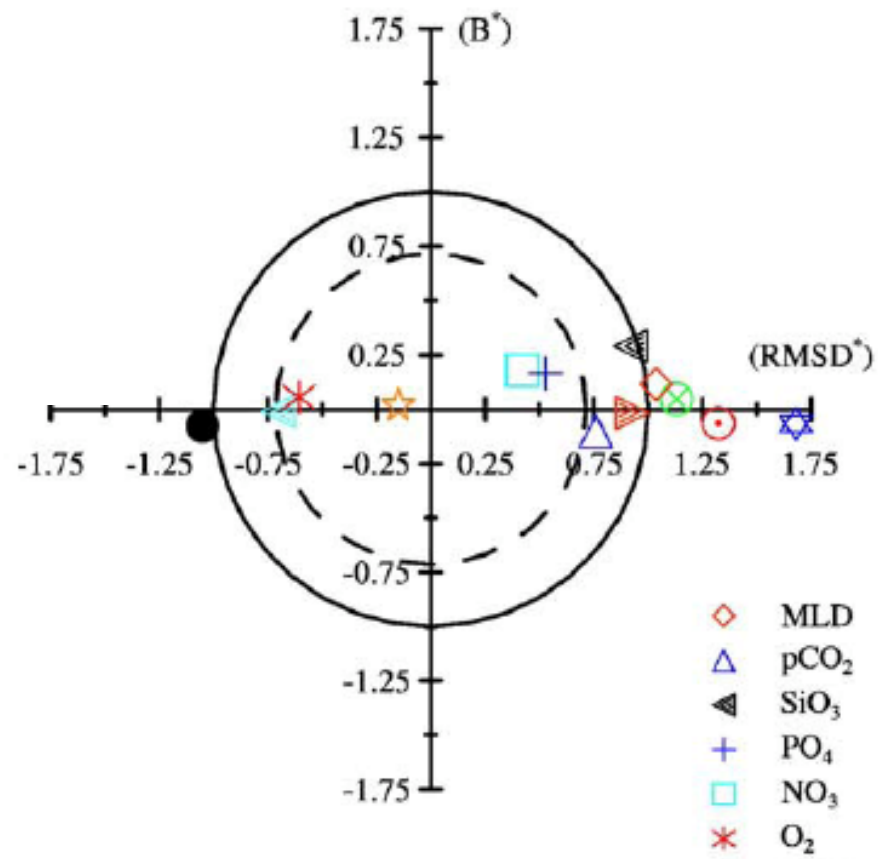


Taylor J. Geophys.  
Res. 2001

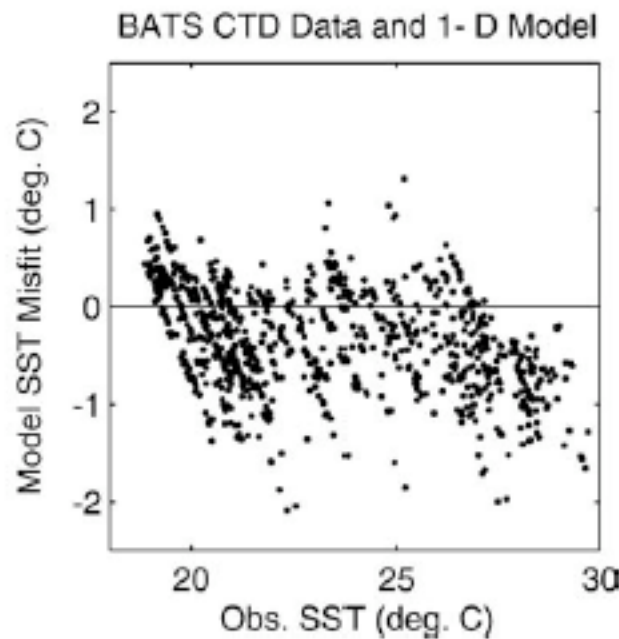
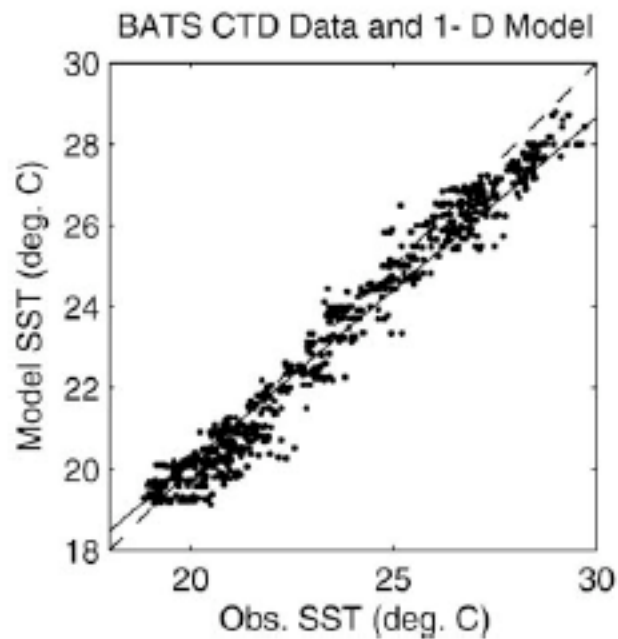
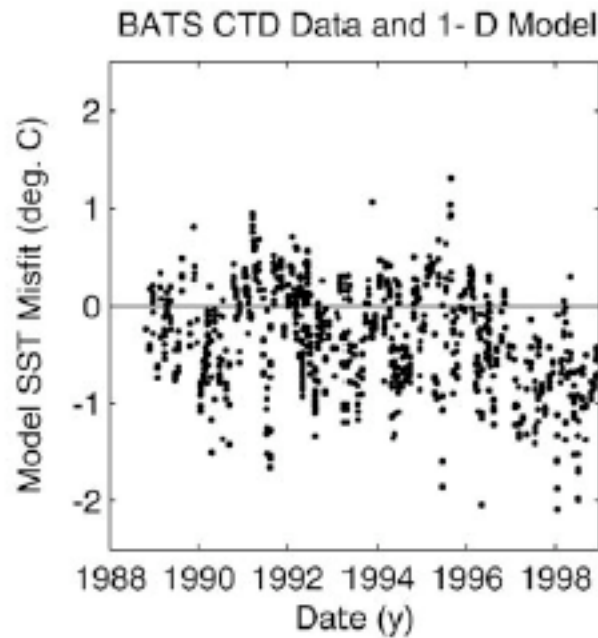
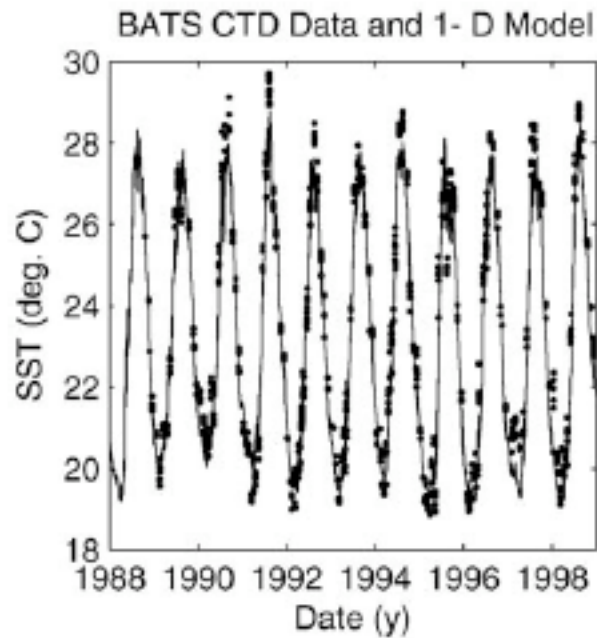
**Figure 2.** Diagram for displaying pattern statistics. The radial distance from the origin is proportional to the standard deviation of a pattern. The centered RMS difference between the test and reference field is proportional to their distance apart (in the same units as the standard deviation). The correlation between the two fields is given by the azimuthal position of the test field.



e-Time (Seasonal Climatology)



Look at the magnitude & structure in model-data residuals



Ducklow et al. Ann.  
Rev. Mar. Res. 2009



## Log-Normal Variables (e.g., chlorophyll)

$$X = \log(\chi) \quad (6)$$

$$\langle \chi \rangle_G = \sqrt[N]{\prod_i \chi_i} = \exp(\langle X \rangle) \quad \text{geometric mean} \quad (7)$$

The geometric bias:

$$\varepsilon_{\text{bias}}^G(\chi) = \exp(\langle X_P \rangle - \langle X_O \rangle) \quad \text{geometric bias} \quad (8)$$

(no bias  $\Rightarrow 1$ )

$$\varepsilon_{\text{rms}}^G(\chi) = \sqrt{\exp\left(\frac{1}{N} \sum (X_P - X_O)^2\right)} \quad \text{geometric rms error} \quad (9)$$

(~normalized to typical data value)

Doney et al. J. Mar. Systems 2009

Stow et al. J. Mar. Systems 2009



3) RI – the reliability index

$$RI = \exp \sqrt{\frac{1}{n} \sum_{i=1}^n \left( \log \frac{O_i}{P_i} \right)^2},$$

average factor  
model differs from  
data

6) MEF – the modeling efficiency

$$MEF = \frac{\left( \sum_{i=1}^n (O_i - \bar{O})^2 - \sum_{i=1}^n (P_i - O_i)^2 \right)}{\sum_{i=1}^n (O_i - \bar{O})^2},$$

predictions relative  
to observed mean

MEF = 1 great  
MEF = 0 no better  
than obs. mean  
MEF < 0 worse  
than obs. mean

$$MEF = 1 - RMSE^2 / s^2$$

$$\chi_v^2 = 1/v \sum_i (P_i - O_i)^2 / \varepsilon_i$$

Reduced Chi  
squared => ~1

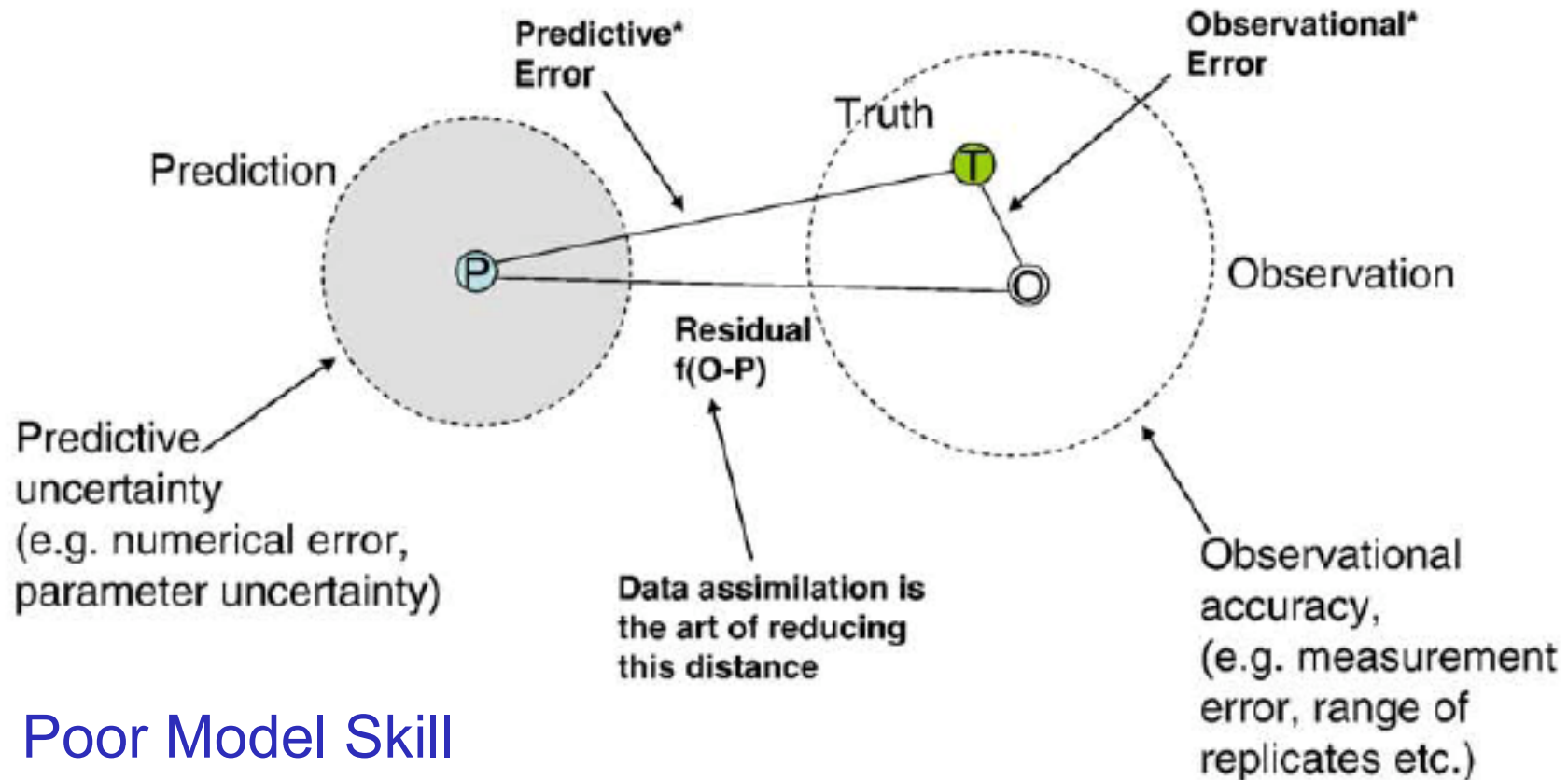
Stow et al. J. Mar. Systems 2009



# Assessing Model Skill

Relationships between the truth, model and data  
(adapted from the ideas of Dan Lynch)

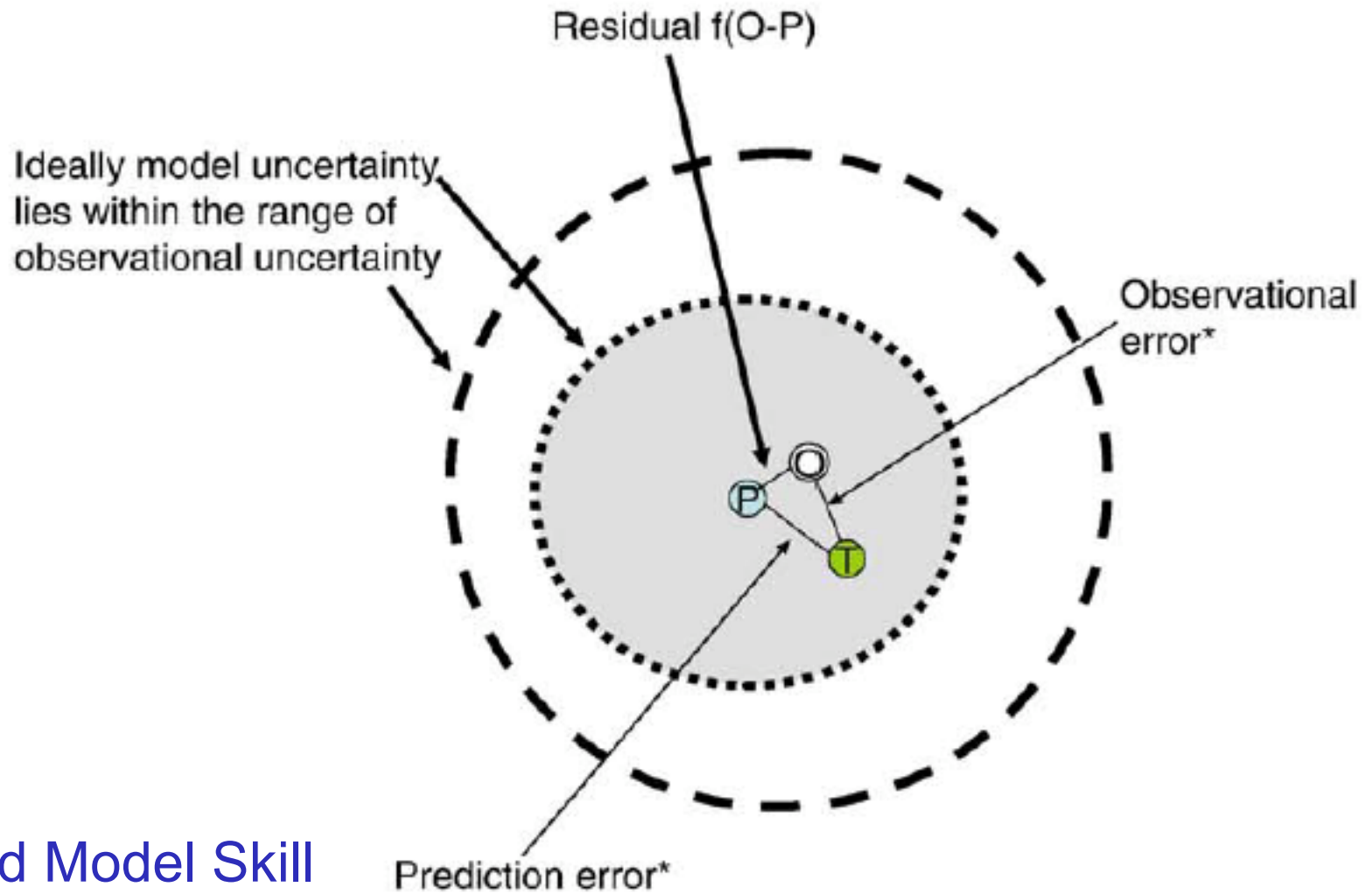
a)



Stow et al. J. Mar. Systems 2009

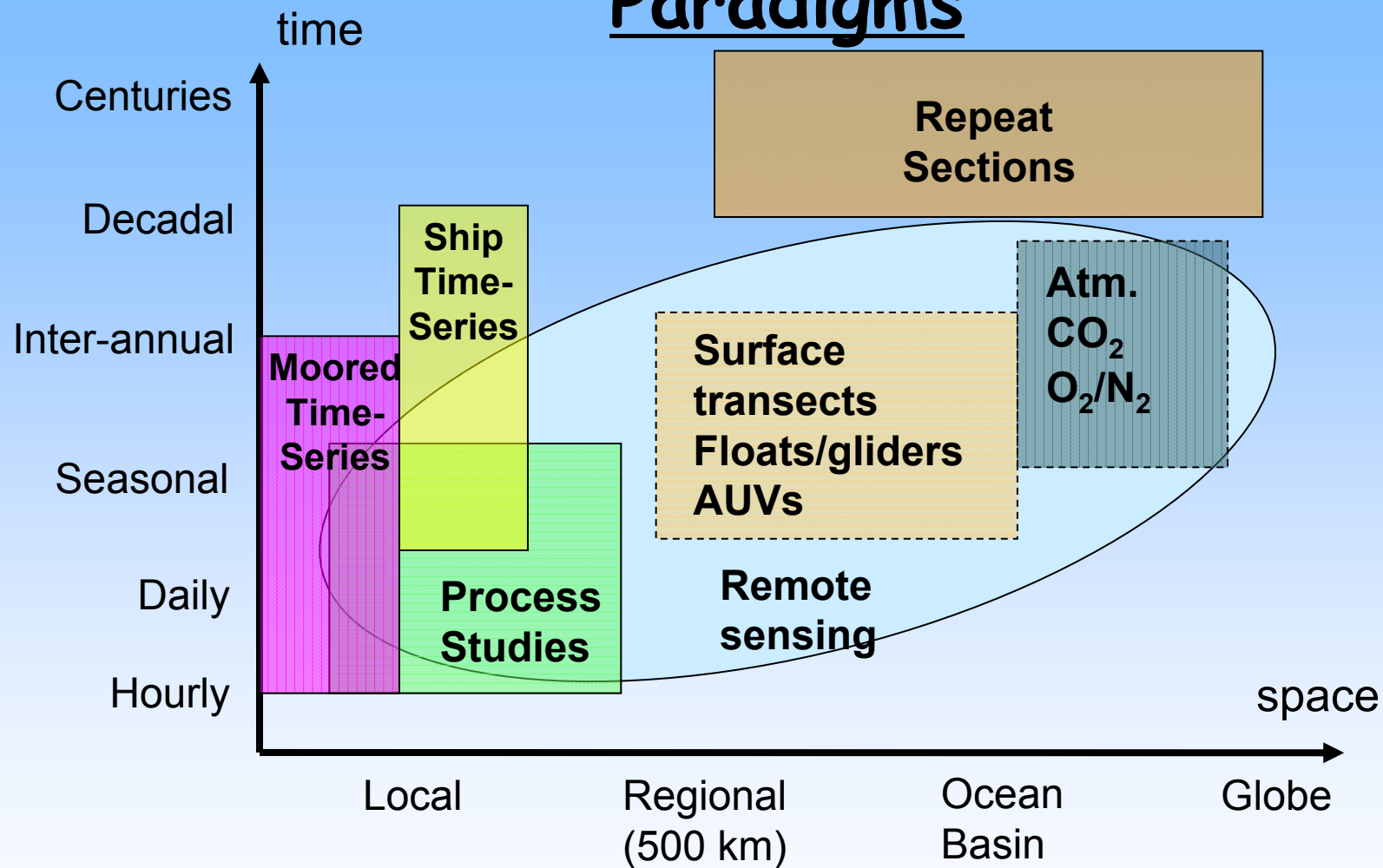


**b)**



Stow et al. J. Mar. Systems 2009

# New Technology & Observational Paradigms



Ocean carbonate system determined by temperature, salinity & 2 of 4 parameters (pH, total carbon, alkalinity, pCO<sub>2</sub>)

Add sensors to autonomous platforms (AUVs, gliders & floats)



# Some Issues to Ponder

## Representativeness of data $y_0$

- “footprint” of observation  
& mismatch with model grid
- local heterogeneity or point sources
- aliasing of unresolved frequencies/wavenumbers  
(e.g., diurnal cycle)
- data selection (i.e., exclude “unrepresentative”  
observations)

$$R = R_{instrument} + R_{representativeness}$$

$$y_{obs} = y_{true} + \epsilon_{obs}$$

$$\epsilon_{obs} = \epsilon_{random} + \epsilon_{systematic}$$

$$E[\epsilon_{obs}^1, \epsilon_{obs}^2] \neq 0$$

# *Modeling Methods for Marine Environments*

David M. Glover, William J. Jenkins &  
Scott C. Doney

- data analysis
- modeling techniques
- ocean examples and applications
- MATLAB based demos and code
- detailed web notes (and perhaps some day a book)

(<http://eos.who.edu/12.747/>)

# Matlab Primer

-can run from Matlab command window or “scripts” (m-files)

-use help & lookfor commands

-define variables (case sensitive) & standard functions:

$a = 7.3e-7$

$b = -\log_{10}(a)$

(follow with “;” if don’t want the answer echoed back)

-vector mathematics

$C=[0:5:100] \Rightarrow C=[0 \ 5 \ 10 \ 15 \ \dots \ 100]$

$C(4) \Rightarrow 15$

$D = 10 * C$

$E = C .* C$  (use “.” for scalar math)

-plotting of 2-D and 3-D graphics

$\text{plot}(C,D,'-')$

# Matlab Primer

-for-loops to cycle over common set of commands

```
for i=1:n
```

```
    F(i) = exp(-i*lambda)
```

```
end
```

-call user-written functions or subroutines

```
C = convert_to_centigrade(F)
```

-hands-on demonstration (m-files)

- Euler vs. 2<sup>nd</sup> order Runge Kutta

- "simple" phytoplankton model

-Ordinary Differential Equation (ODEs) integrators

- find  $y(t)$  from  $y(t_0)$  and equation for  $dydt=f(y,t,p)$

- define "function" to integrate e.g. "dydt" (m-file)

```
[T,Y] = ode12s('dydt',T,Y0)
```

CO<sub>2</sub> thermodynamics code  $[pH,pCO_2, \dots]=f(DIC,ALK,T,S\dots)$