An Introduction to Radiative Transfer in the Earth System

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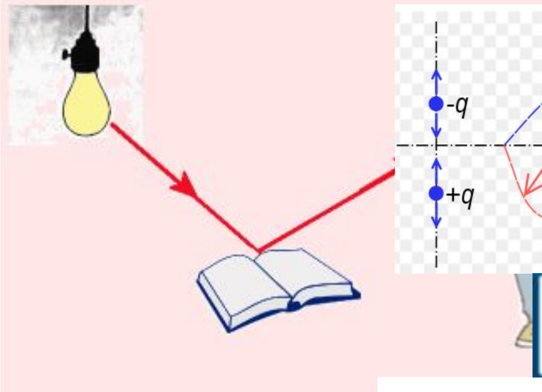
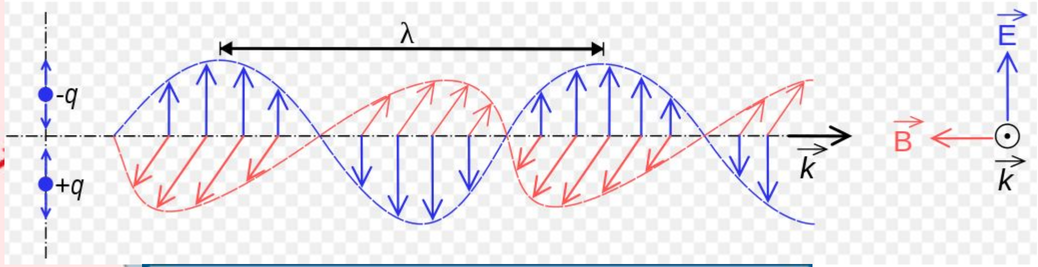
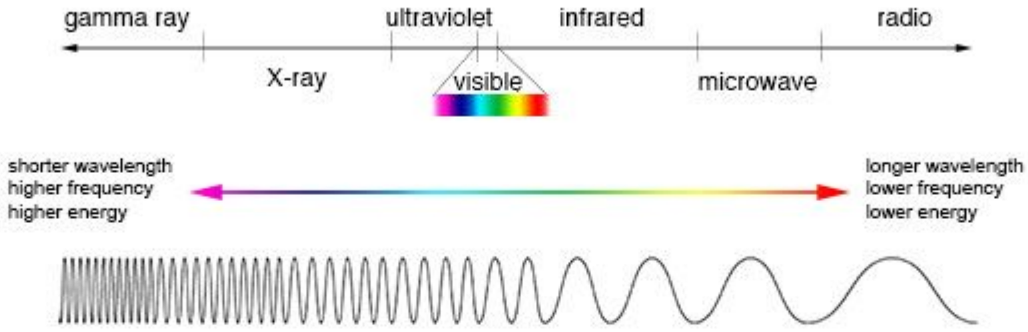
PACE Workshop, Sponsor: OSB

Why can you see the world?



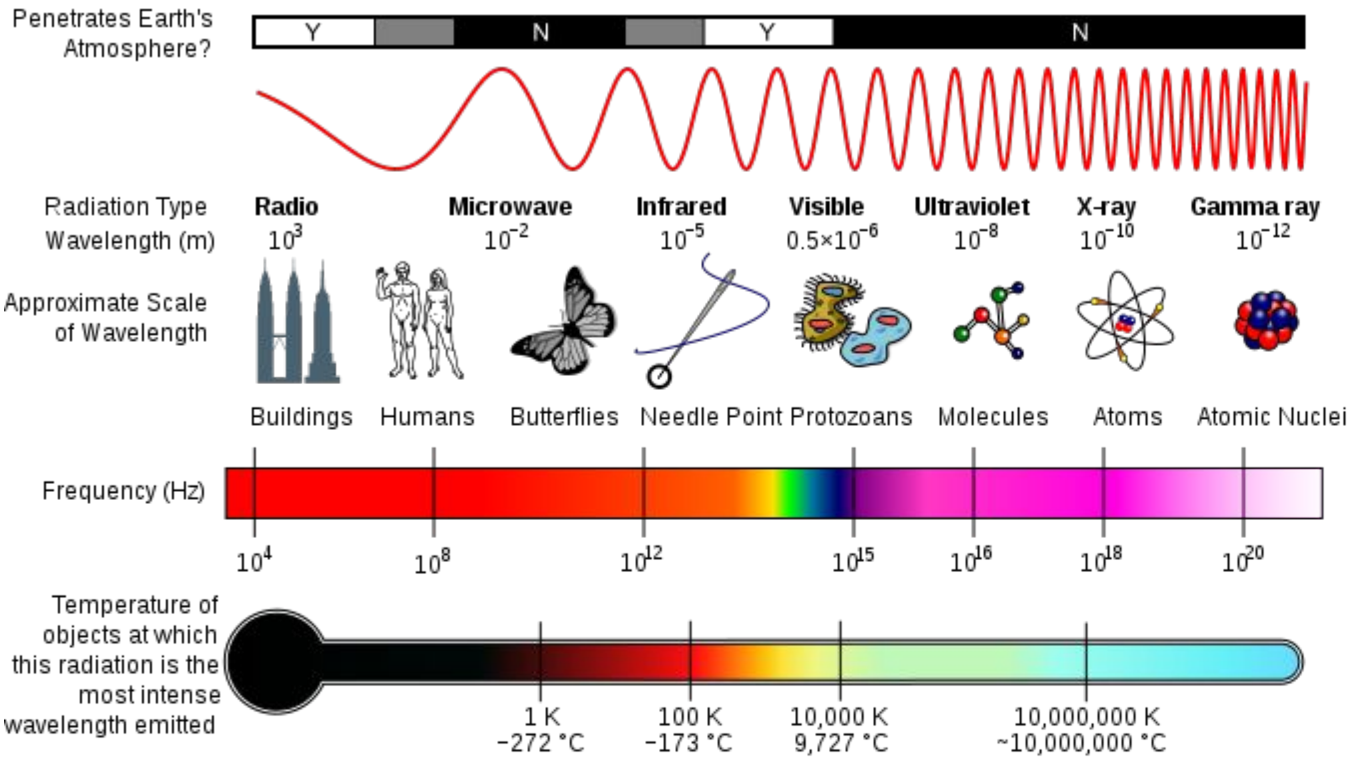


$$\mathbf{E}(z,t) = \mathbf{E}_0 \exp(ikz - i\varpi t)$$



Electromagnetic Spectrum

$$\lambda = v / f$$



Stokes Parameters

The electric field can be resolved into two components as follows: $$\mathbf{E} = E_{\parallel}\hat{\mathbf{e}}_{\parallel} + E_{\perp}\hat{\mathbf{e}}_{\perp}$$

Where $E_{\parallel}$ and $E_{\perp}$ are components parallel and perpendicular to **a reference plane**, respectively.

The four component Stokes vector can now be defined as:

$$I = E_{\parallel}E_{\parallel}^{*} + E_{\perp}E_{\perp}^{*} = I_{\parallel} + I_{\perp}$$

What human eyes can see.

$$Q = E_{\parallel}E_{\parallel}^{*} - E_{\perp}E_{\perp}^{*} = I_{\parallel} - I_{\perp}$$

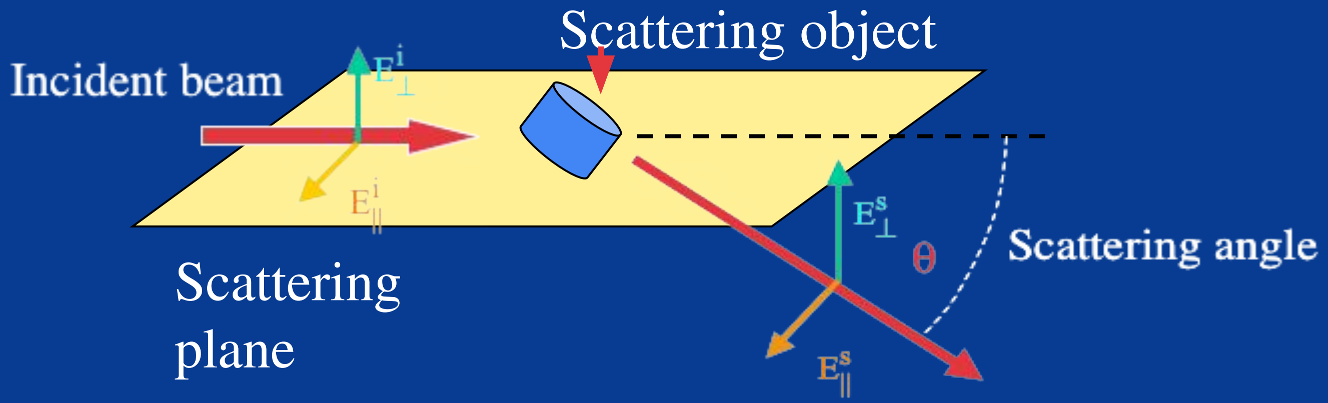
$$U = E_{\parallel}E_{\perp}^{*} + E_{\perp}E_{\parallel}^{*}$$

Linear polarization.

$$V = i\left(E_{\parallel}E_{\perp}^{*} - E_{\perp}E_{\parallel}^{*}\right)$$

Circular polarization.

Light Scattering Geometry



Amplitude scattering matrix

$$\begin{pmatrix} E_\parallel^s \\ E_\perp^s \end{pmatrix} = \frac{e^{ikr}}{-ikr} \begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} \begin{pmatrix} E_\parallel^i \\ E_\perp^i \end{pmatrix}$$

Light Scattering: Mueller Matrix

$$\begin{pmatrix} I^s \\ Q^s \\ U^s \\ V^s \end{pmatrix} = \frac{1}{k^2 r^2} \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} I^i \\ Q^i \\ U^i \\ V^i \end{pmatrix}$$

$$M_{11} = \frac{1}{2}(|S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2)$$

$$M_{12} = \frac{1}{2}(|S_2|^2 - |S_1|^2 + |S_4|^2 - |S_3|^2)$$

$$M_{13} = \mathrm{Re}\{S_2 S_3^* + S_1 S_4^*\}$$

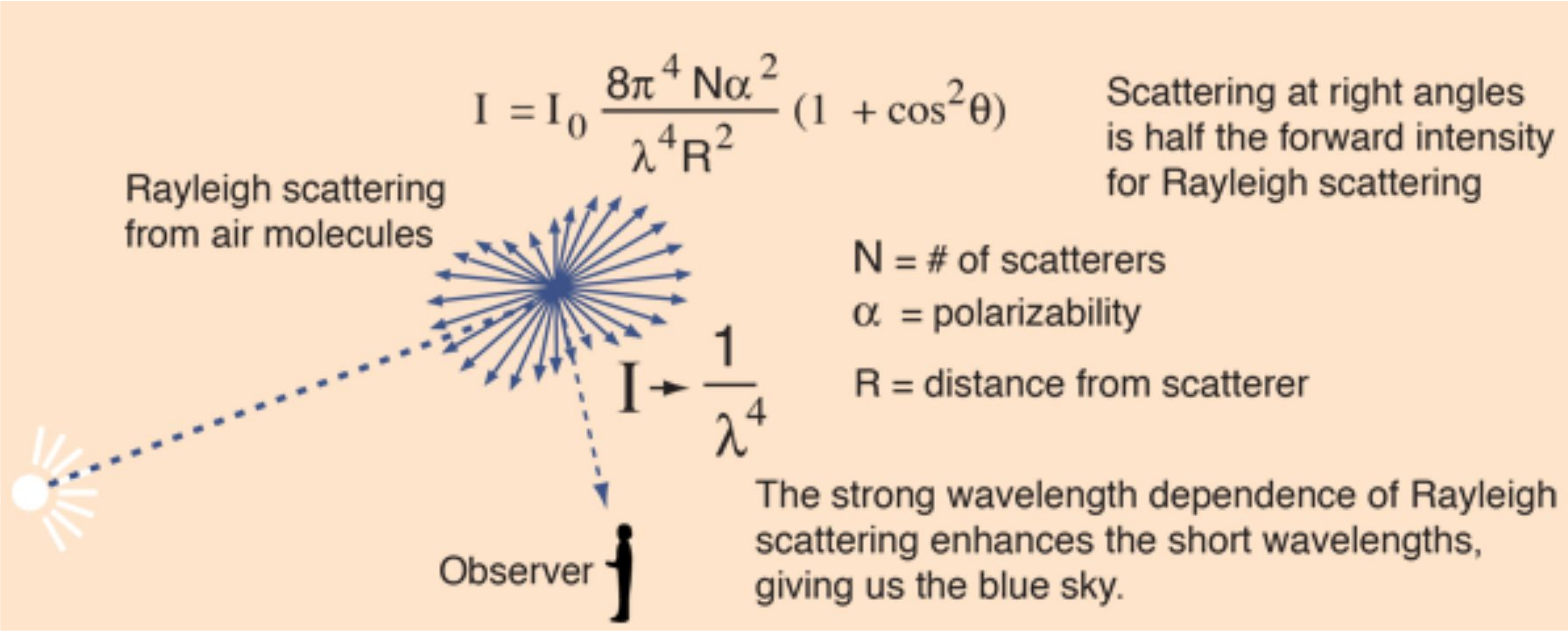
$$M_{14} = \mathrm{Im}\{S_2 S_3^* - S_1 S_4^*\}$$

$$p(\theta) = \frac{4\pi}{C_{scat}} M_{11}(\theta)$$

$$C_{ext} = C_{scat} + C_{abs}$$

$$\omega = \frac{C_{scat}}{C_{ext}}$$

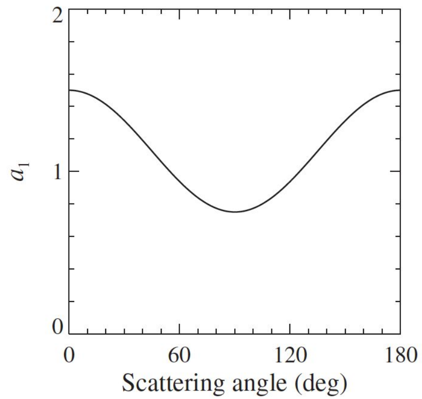
Light Scattering by Molecules: Rayleigh Scattering



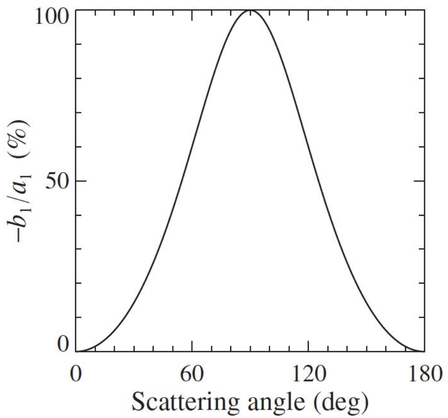
Source: http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html

Light Scattering by Molecules: Scattering Matrix

$$\mathbf{P}(\Theta) = \frac{3}{2}\frac{(1-\rho)}{(1+\rho/2)}\begin{bmatrix} \frac{1}{2}\left(\frac{1+\rho}{1-\rho}+\cos^2\Theta\right) & -\frac{1}{2}(1-\cos^2\Theta) & 0 & 0 \\ -\frac{1}{2}(1-\cos^2\Theta) & \frac{1}{2}(1+\cos^2\Theta) & 0 & 0 \\ 0 & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & \frac{(1-2\rho)}{(1-\rho)}\cos\Theta \end{bmatrix}$$



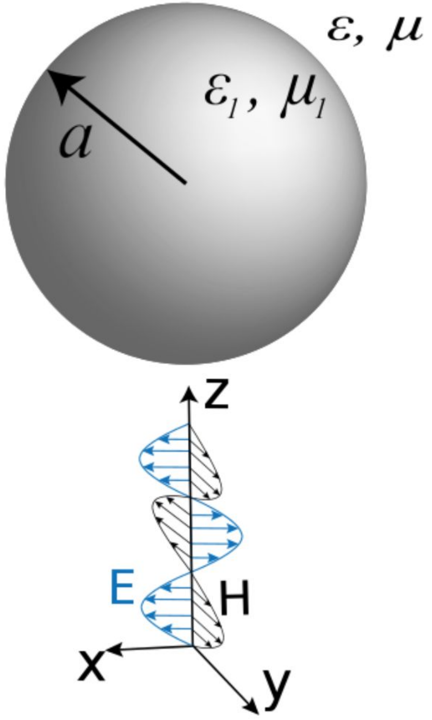
The depolarization factor ρ has a value equal to the ratio of the perpendicular and parallel scattered intensities at right angles, if the incident is unpolarized. The average value of ρ is 0.028 for airs (Thomasi, et al., 2005).



Souce: https://www.ess.uci.edu/~cmclinden/link/xx/node21.html

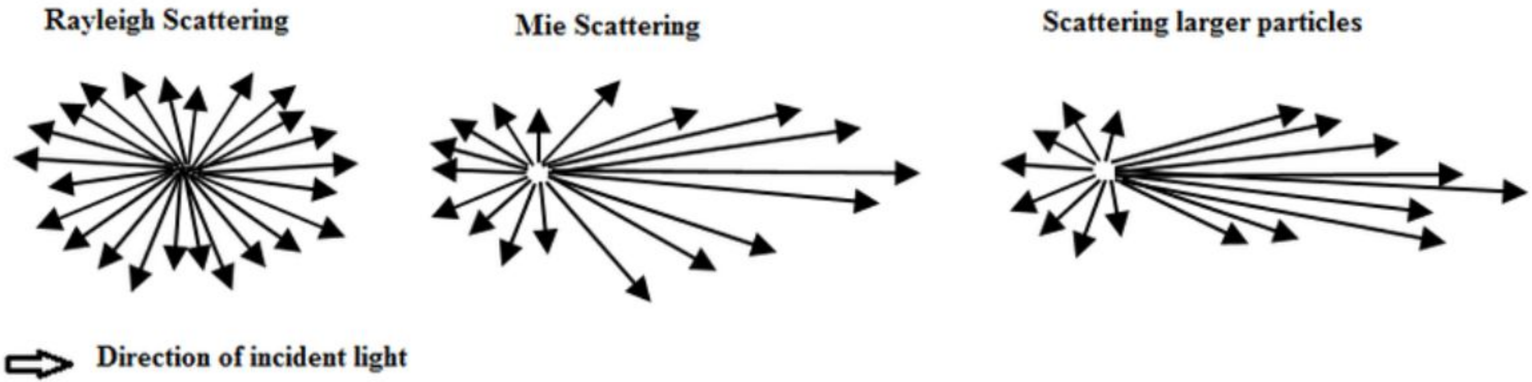
Light scattering by spherical particles: Lorenz–Mie Theory

- Lorenz–Mie theory solves scattering of plane waves by a spherical particles characterized by permittivity ϵ and permeability μ.
- In physics this problems can be formulated as solving the Helmholtz equation: $\nabla^2\vec{E} + k^2\vec{E} = 0$, $\nabla^2\vec{H} + k^2\vec{H} = 0$
- All three waves: incident, scattering, and internal waves are expanded as a linear combination of vector spherical harmonics.
- The expansion coefficients are obtained by using the matching conditions of the electromagnetic fields.
- Scattering properties: scattering, absorption, extinction cross sections, and scattering matrix are determined by the expansion coefficients.



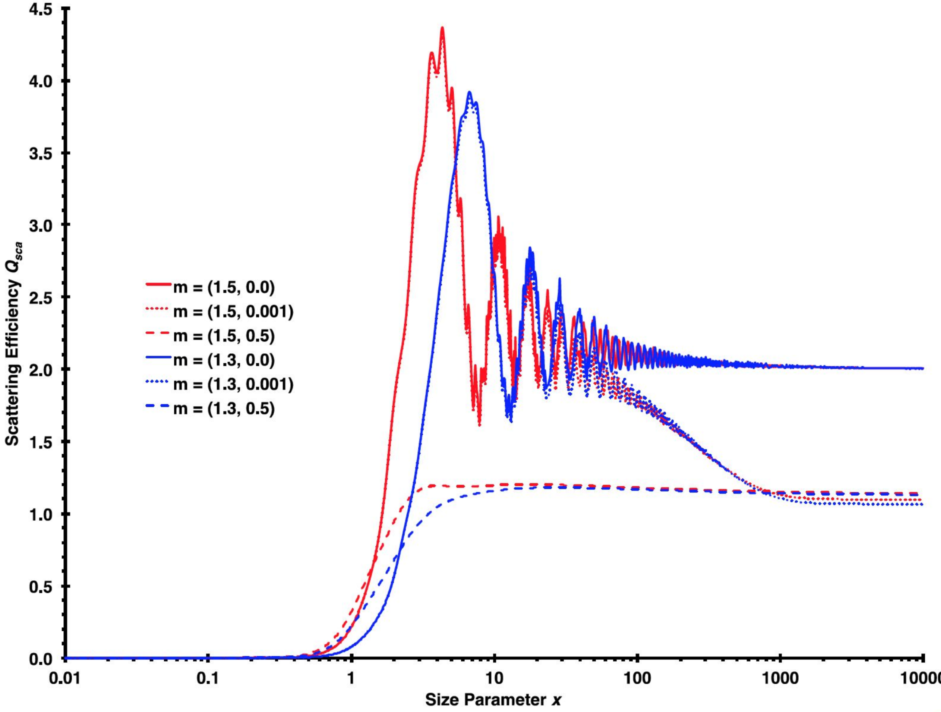
Mie Scattering Characteristics

- The scattering properties of spherical particle is determined by the size parameter $X = 2\pi a/\lambda$ and the relative refractive index n.
- When X increase, the forward scattering increases.



Scattering efficiency: $Q_{scat} = C_{scat}/\pi a^2$

- Q_{scat} increases as X increases when X is small (<2).
- Then Q_{scat} enters an oscillatory region as X increases.
- If no absorption, $Q_{scat} \rightarrow 2$ when $X \rightarrow \infty$.
- If there is absorption (imaginary refractive index is non-zero, $Q_{scat} \rightarrow 1$ when $X \rightarrow \infty$.



Source: Moosmüller and Ogren, MDPI Atmosphere, 2017

Available Mie Codes (a noncomplete list)

- Wiscombe, 1980. https://github.com/cfinch/Mie_scattering/tree/master/Wiscombe
- Mishchenko,
  - http://www.giss.nasa.gov/~crmim/books.html
  - https://www.giss.nasa.gov/staff/mmishchenko/ftpcode/spher.f
- https://scattport.org/index.php/light-scattering-software

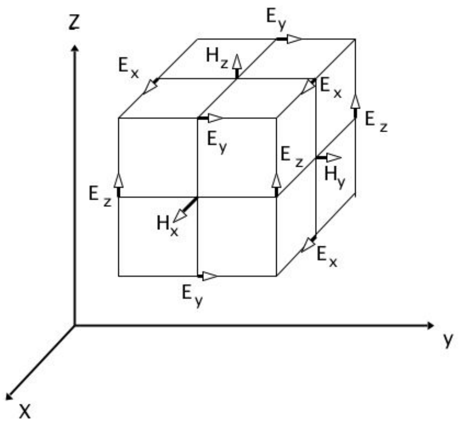
Scattering by non-spherical particles: FDTD method

FDTD: Finite Difference Time Domain

Maxwell Equations:

$$\nabla \times H = \frac{\varepsilon}{c}\frac{\partial E}{\partial t}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}$$



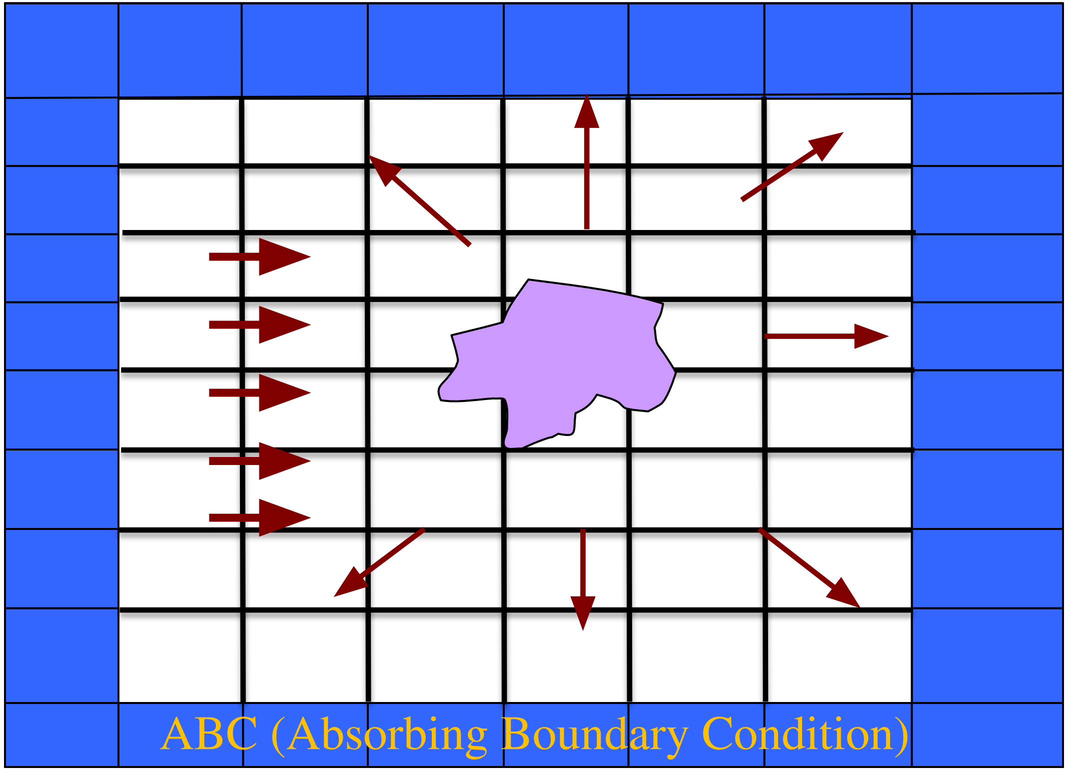
- Spatial discretization using Yee algorithm.

$$\frac{\partial u(i\Delta x, j\Delta y, k\Delta z, n\Delta t)}{\partial x} = \frac{u^n_{i+1/2,j,k} - u^n_{i-1/2,j,k}}{\Delta x} + O\left[(\Delta x)^2\right]$$

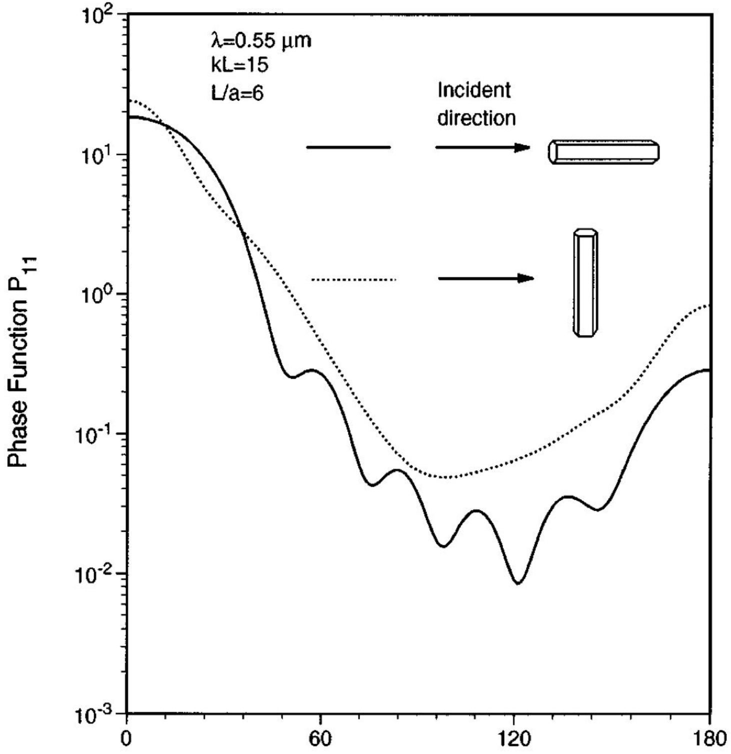
$$\frac{\partial u(i\Delta x, j\Delta y, k\Delta z, n\Delta t)}{\partial t} = \frac{u^{n+1/2}_{i,j,k} - u^{n-1/2}_{i,j,k}}{\Delta t} + O\left[(\Delta t)^2\right]$$

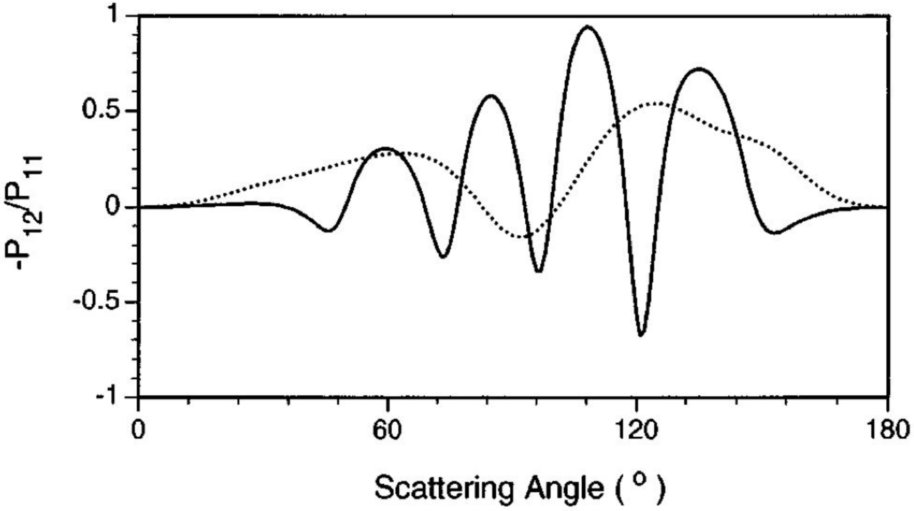
P. Yang and K. N. Liou, J. Opt. Soc. Am. A **12**, 162-176 (1995)
W. Sun, et al., Appl. Opt. **38**, 3141-3151 (1999)

FDTD Method Cont.



Scattering by non-spherical particles: FDTD method

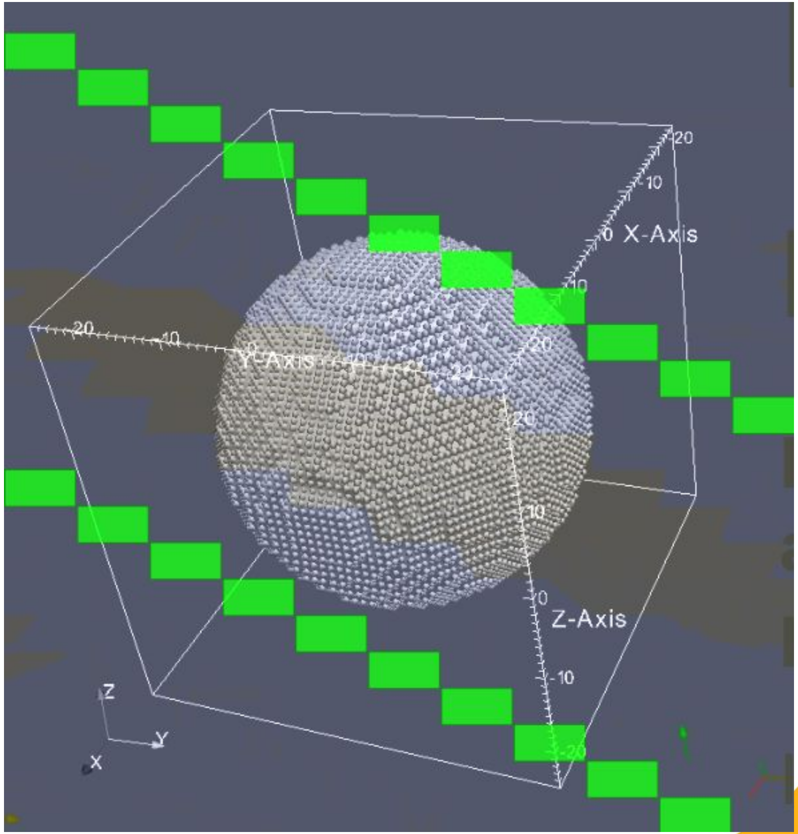




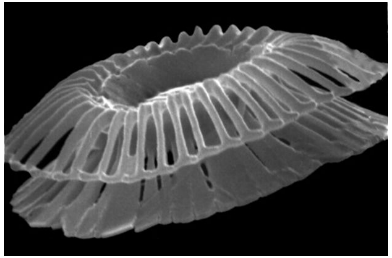
Source: **Yang and Liou,** J. Opt. Soc. Am. A/Vol. 13, **1996**

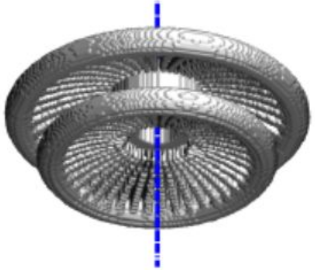
Scattering by non-spherical particles: DDA Method

- Scattering objects are discretized into small dipoles.
- Each dipole is influenced by the incident wave and the scattering wave by other dipoles.
- The scattering of each dipole is solvable, thus we can write the whole scattering system as a system of linear equations
- Codes:
  - http://ddscat.wikidot.com/start
  - https://github.com/adda-team/adda

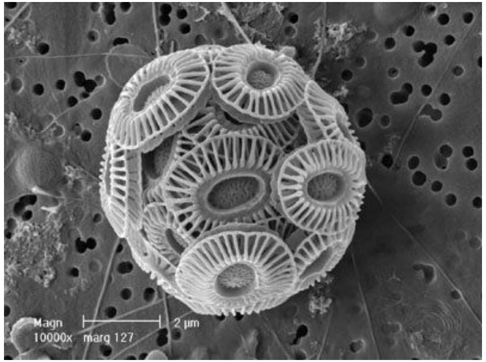


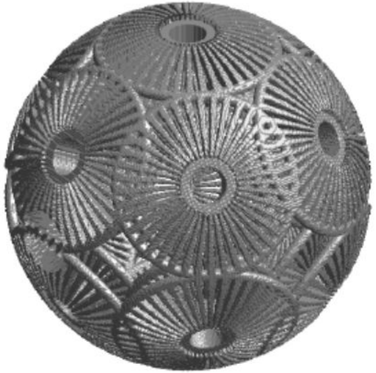
Using DDA for Non-spherical Particles: Coccolithophore





The Amsterdam DDA Solver is used for solving the light scattering.





Peng-Wang Zhai, et al., "Inherent optical properties of the coccolithophore: Emiliania huxleyi," Opt. Express 21, 17625-17638 (2013)

Scattering by non-spherical particles: T-matrix Method

- Similar to the Mie theory, T-matrix method expands the incident, scattering, and internal waves into vector spherical harmonics.
- The expansion coefficients are found by using matching conditions of the electromagnetic fields (tangential components are continuous).
- Theoretically it can be used for arbitrarily shaped particles. In reality it is best used for particles with certain symmetrical properties, for instance, spheroids, ellipses, etc., for better convergence and performance.
- For more info: https://www.giss.nasa.gov/staff/mmishchenko/t_matrix.html

Summary of Light Scattering Solvers

- Spherical particle of arbitrary size: Lorenz Mie theory.
- Non-spherical particles:
  - FDTD method (Yang and Liou, 1995, 1996, Sun et al., 1999)
  - DDA method (Draine and Flatau, 1994, Yurkin and Hoekstra, JQSRT, 2011)
  - T-matrix (Mishchenko, et al., , JQSRT, 1996.)
- Other methods:
  - Finite element
  - Point matching
  - Geometric optics (Ray tracing)
  - Invariant Imbedding T-matrix method (Bi et al., 2012)

Light scattering by a collection of particles: size distribution

- the modified gamma distribution

$$n(r) = \text{constant} \times r^{\alpha} \exp\left(-\frac{\alpha r^{\gamma}}{\gamma r_c^{\gamma}}\right);$$

- the log normal distribution

$$n(r) = \text{constant} \times r^{-1} \exp\left[-\frac{(\ln r - \ln r_g)^2}{2 \ln^2 \sigma_g}\right];$$

- the power law distribution

$$n(r) = \begin{cases} \text{constant} \times r^{-3}, & r_1 \le r \le r_2, \\ 0, & \text{otherwise}; \end{cases}$$

- the gamma distribution

$$n(r) = \text{constant} \times r^{(1-3b)/b} \exp\left(-\frac{r}{ab}\right), \quad b \in (0, 0.5);$$

$$r_{\text{eff}} = \frac{1}{\langle G \rangle} \int_{r_{\min}}^{r_{\max}} \mathrm{d}r\, n(r) r \pi r^2,$$

$$v_{\text{eff}} = \frac{1}{\langle G \rangle r_{\text{eff}}^2} \int_{r_{\min}}^{r_{\max}} \mathrm{d}r\, n(r)(r - r_{\text{eff}})^2 \pi r^2,$$

$$\langle C_{\text{ext}} \rangle = \int_{r_{\min}}^{r_{\max}} \mathrm{d}r\, n(r) C_{\text{ext}}(r)$$

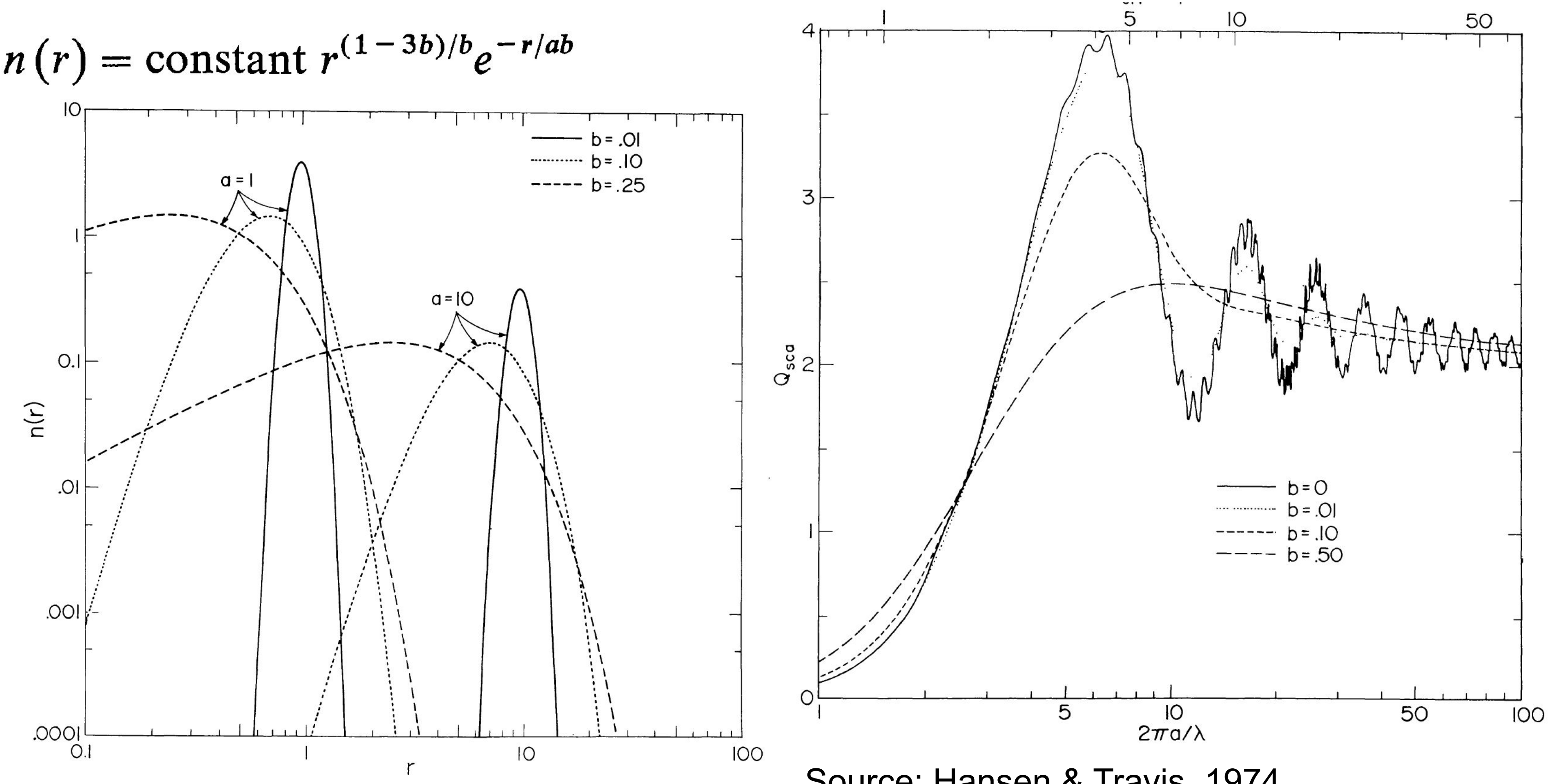
$$\langle C_{\text{sca}} \rangle = \int_{r_{\min}}^{r_{\max}} \mathrm{d}r\, n(r) C_{\text{sca}}(r)$$

$$\langle \mathbf{P}(\Theta) \rangle = \frac{1}{\langle C_{sca} \rangle} \int_{r_{min}}^{r_{max}} dr \mathbf{P}(r, \Theta) n(r) C_{sca}(r)$$

Source: Mishchenko et al., 2002

Effects of size distribution integration

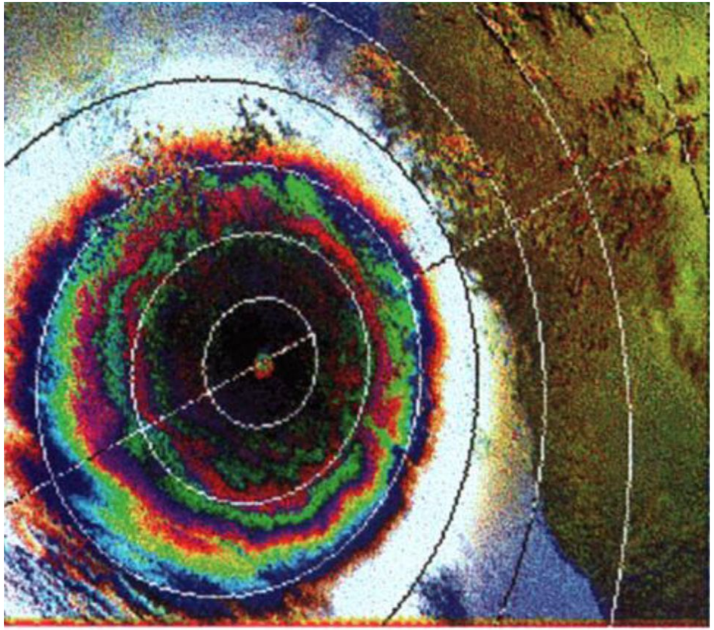
$$n(r) = \text{constant}\ r^{(1-3b)/b} e^{-r/ab}$$



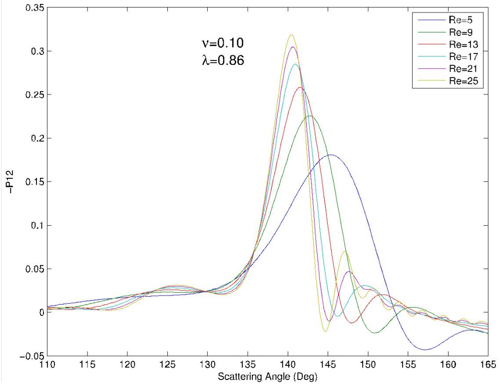
Source: Hansen & Travis, 1974

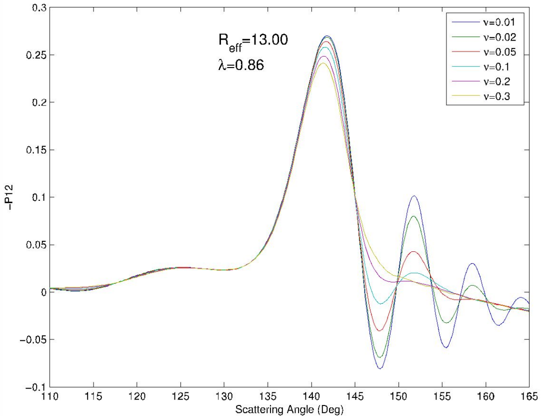
Rainbow as a Tool for Remote Sensing

Image credit: national geographic



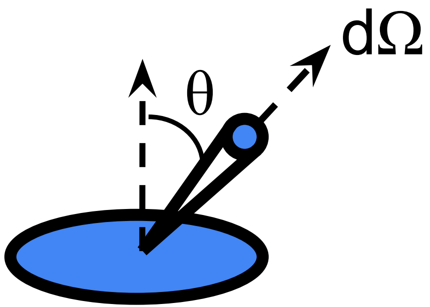
Three color composite (Blue: 0.43 gm, Green: 0.67 grn, Red: 0.86 grn) of the polarized POLDER measurements over the Atlantic ocean and Southern Africa on Nov. 3rd, 1996. Source: Bréon and Goloub, 1996.





Section II: Radiative Transfer

Stokes Parameters: Phenomenological Definitions



$$\mathbf{I}_\lambda = \frac{dE_\lambda}{\cos(\theta)\, d\sigma\, d\Omega\, d\lambda\, dt}$$

$$F_\lambda = \int I_\lambda \cos(\theta)\, d\Omega$$

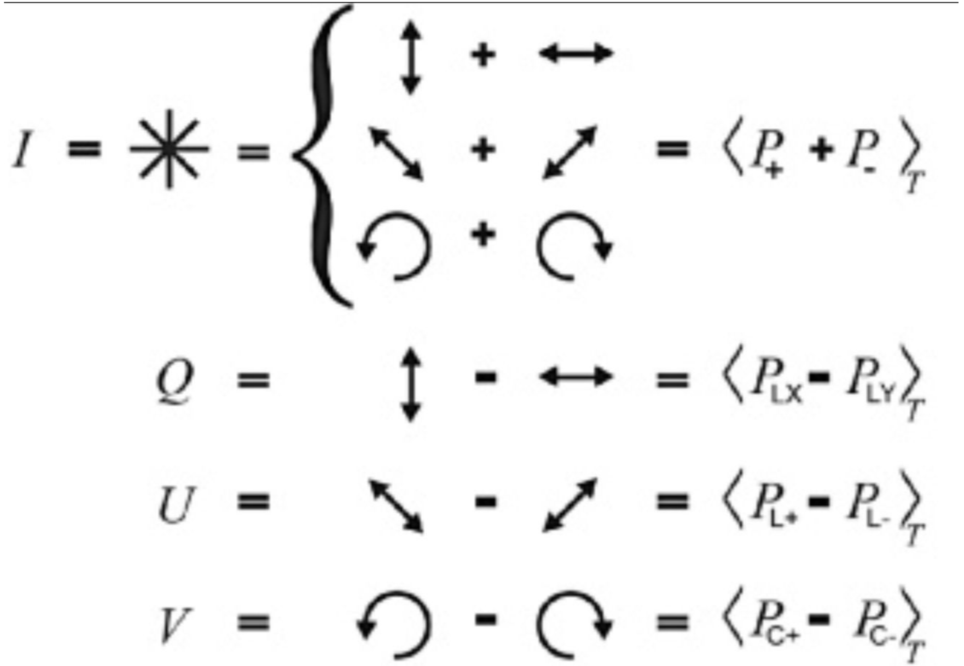


Image credit: Akarçay, et al., "Monte Carlo modeling of polarized light propagation: Stokes vs. Jones. Part I," Appl. Opt. 53, 7576-7585 (2014)

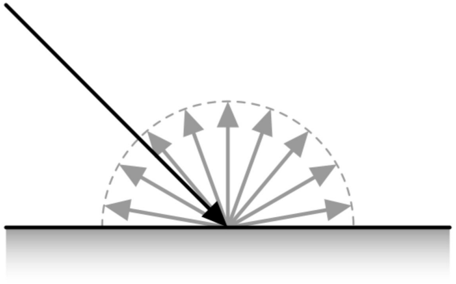
Interaction of a Beam of Light with a Turbid Matter

- Scattering and absorption by particles (Molecules, Aerosols, Clouds, Hydrosols, etc.)
- Reflection by surfaces (land and ocean surfaces)
- Absorption by gas molecules

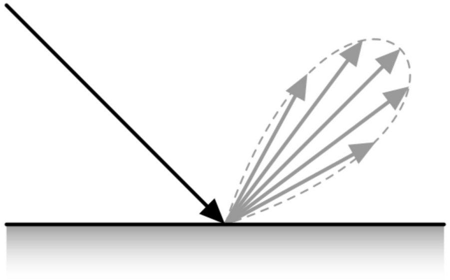
Surface Reflection: BRDF

- BRF: bidirectional reflectance factor (BRF): $R(\Omega, \Omega') = \frac{\pi I(\Omega)}{\mu F(\Omega')}$
- BRDF: bidirectional reflectance distribution function: $\mathrm{B}(\Omega, \Omega') = \frac{I(\Omega)}{\mu F(\Omega')}$
- $F(\Omega')$ is downwelling irradiance measured with a sensor perpendicular to Ω'.
- $\mu = \cos\theta$
- $\Omega' = (\theta', \phi')$, incident angles
- $\Omega = (\theta, \phi)$, reflection angles.
- Wavelength dependence is implicitly assumed.

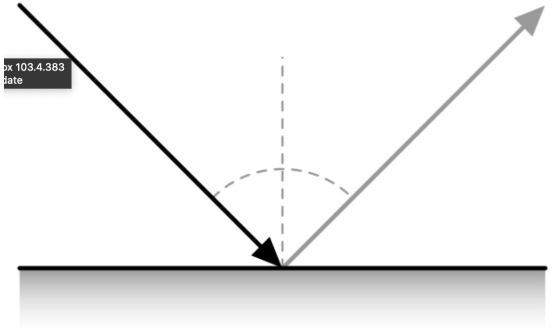
Different types of reflection surface



Lambertian (cosine) reflection.



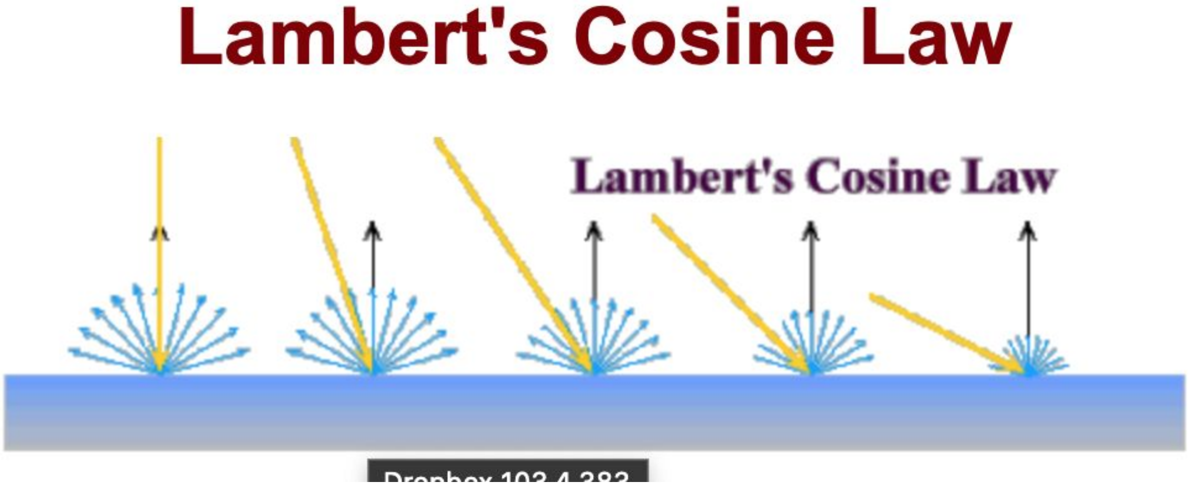
Glossy



Mirror reflection.

Image source: https://en.wikipedia.org/wiki/Bidirectional_reflectance_distribution_function

Lambertian Reflectance

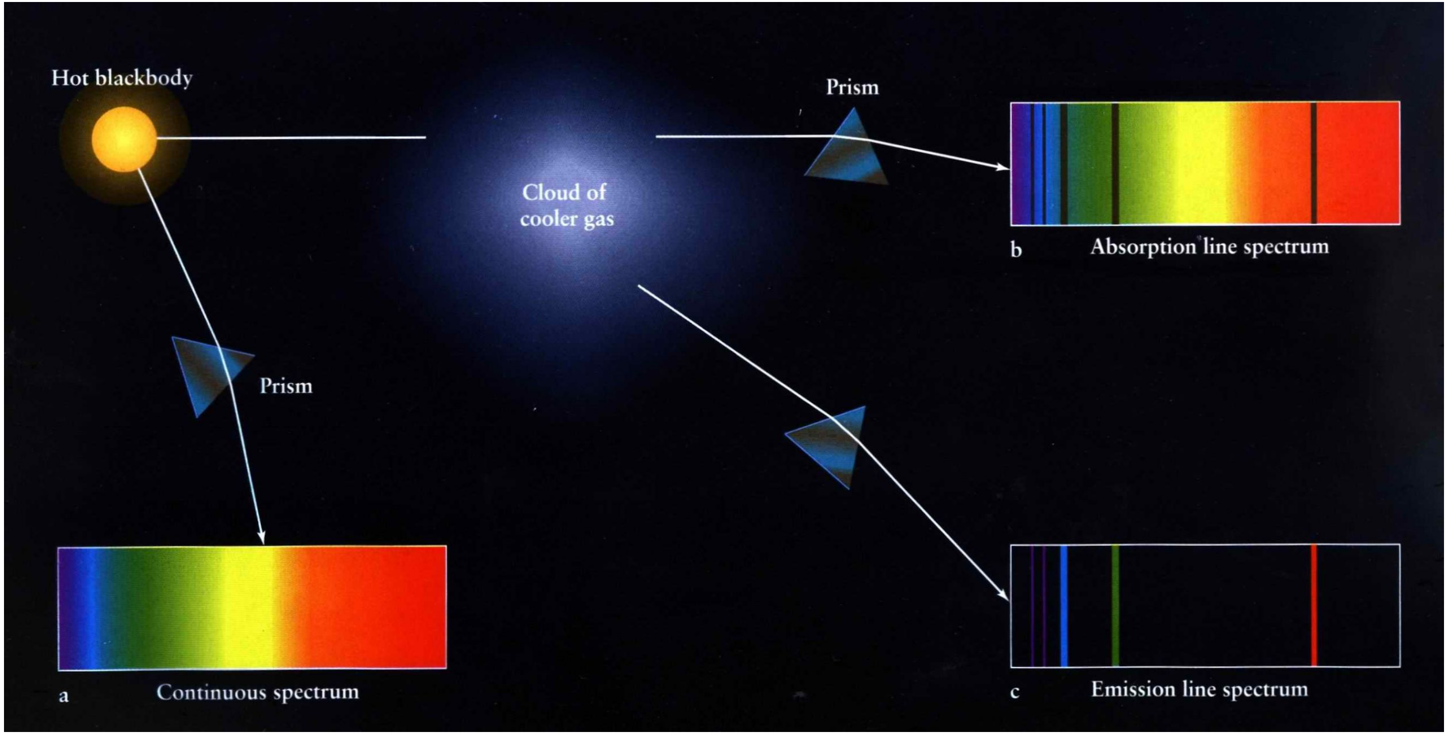


Law : reflected energy from a small surface area in a particular direction is proportional to the cosine of the angle between that direction and the surface normal

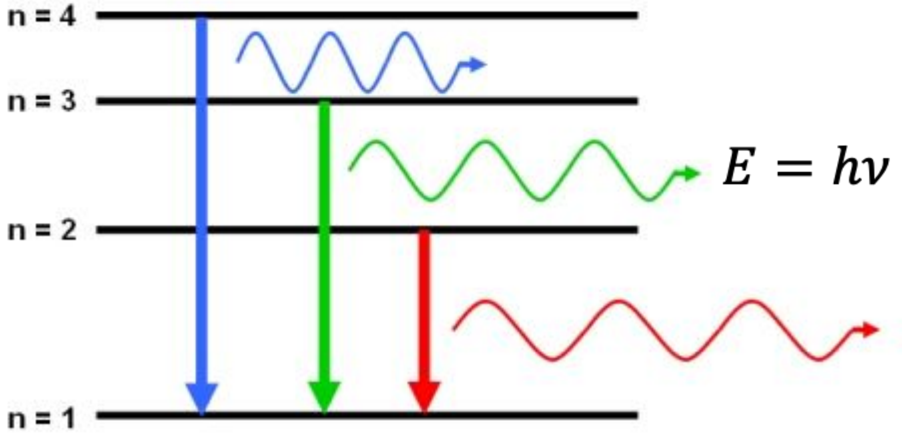
Reflected intensity is independent of the viewing direction, but dependent of the source orientation.

Source: http://www.vrarchitect.net/anu/cg/Illumination/lambertCosineLaw.en.html

Gas absorption line



Spectral line broadening mechanisms



$$\mathbf{E = E_{rot} + E_{vib} + E_{el} + E_{tr}}$$

For each energy transition, the photon frequency ν is not monochromatic due to:

- Natural broadening.
- Pressure (collision) broadening.
- Doppler broadening.

Natural Broadening

- Heisenberg's uncertainty principle:

$$\Delta E \cdot \Delta t = \frac{h}{2\pi}$$

where $h = 6.626 \cdot 10^{-34}\, J \cdot s$ is the Planck constant.

- $\Delta \nu_N = \frac{\Delta E}{h} = \frac{1}{2\pi \Delta t_N}$
- Finite atomic or molecule state life time Δt leads to finite frequency uncertainty $\Delta \nu$

Natural Broadening Cont.

- The population n of the molecule/atom at upper state:

$$\frac{dn(t)}{dt} = -An(t),$$

A is the Einstein A coefficient. The solution is $n(t) = n(0)\exp(-At) = n(0)\exp(-t/\tau)$.
τ is the mean lifetime of the excited state.
The number of spontaneously emitted photons would be proportional to $n(t)$. Therefore we can write the flux F that:

$$F(t) = F(0)\exp(-t/\tau) = F(0)\exp(-\gamma t)$$

A Fourier transform of this function lead to the frequency shape function:

$$F(\nu) = \frac{1}{\pi}\frac{\gamma/4\pi}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

Which is the Lorentzian profile.

Pressure broadening

- The life time of an atomic/molecular state is easily perturbed by collisions with other atom/molecule.
- Due to Heisenburg's uncertainty principle, this leads to additional frequency uncertainty:

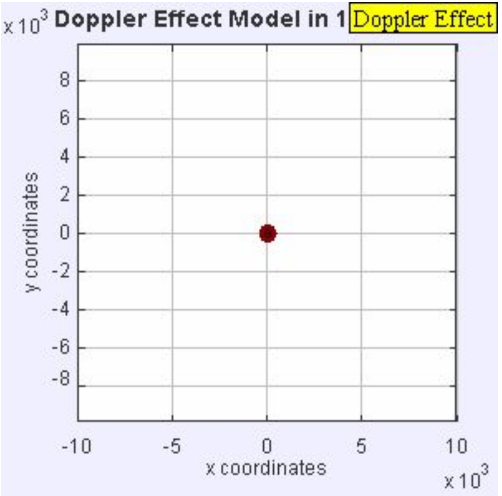
$$\Delta\nu_p = \frac{1}{2\pi\Delta t_p}$$

$$F_L(\nu) = \frac{\gamma_L}{\pi}\frac{1}{(\nu-\nu_0)^2+\gamma_L^2}$$

This is quite similar to the natural broadening spectral shape.
However, be warned that the parameter value is different.

$$\gamma_L \sim P \cdot T^{-1/2}$$

Doppler Broadening



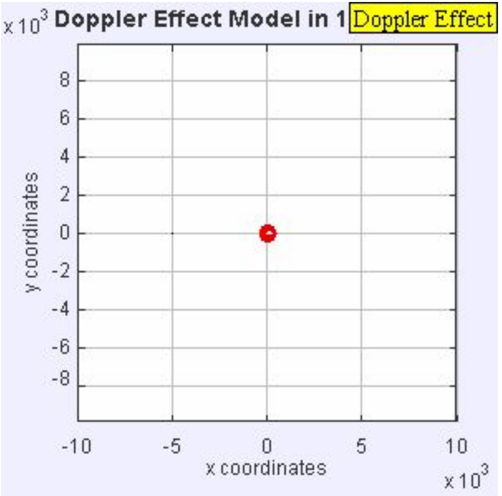


Image credits: Lookang many thanks to Fu-Kwun Hwang and author of Easy Java Simulation = Francisco Esquembre

$$\nu_m = \nu_0 \left(1 + \frac{\Delta v}{c}\right)$$

Speed of source relative to observer

Speed of light

Measured Frequency

Frequency for a stationary source

Doppler Broadening cont.

- Gaussian velocity distribution function (leads to Gaussian $\Phi(\Delta v)$):

$$\phi(\nu) = \underbrace{\frac{2\sqrt{\ln 2}}{\sqrt{\pi}\Delta\nu_D}}_{\phi(\nu_0)} \exp\left[-\left(\frac{2\sqrt{\ln 2}}{\Delta\nu_D} (\nu - \nu_0) \right)^2 \right]$$

$$\Delta\nu_D(\mathrm{FWHM}) = 2\sqrt{\frac{2kT\ln 2}{mc^2}}\nu_0$$

$$\Delta\nu_D(\mathrm{FWHM}) = 7.17 \times 10^{-7} \nu_0 \sqrt{\frac{T}{M}}$$

g/mole of emitter/absorber

Source: https://cefrc.princeton.edu/sites/cefrc/files/Files/2013%20Lecture%20Notes/Hanson/pLecture6.pdf

- Doppler broadening most significant at:

  Low P, high T, small λ

- Collision broadening most significant at:

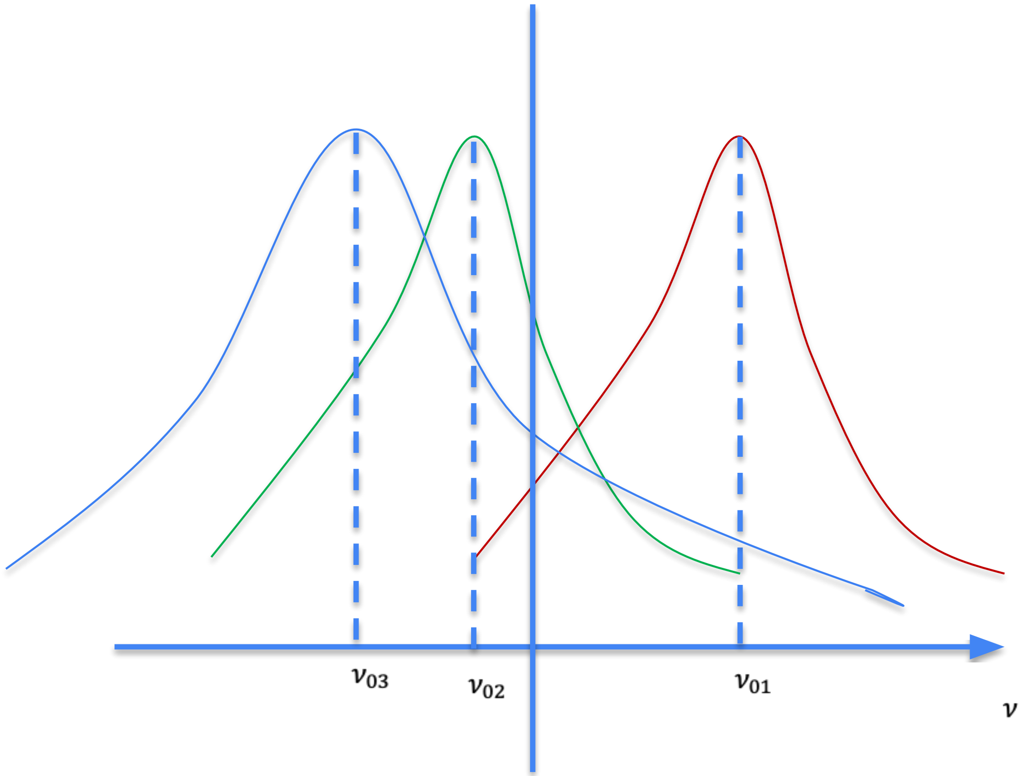
  High P, low T, large λ

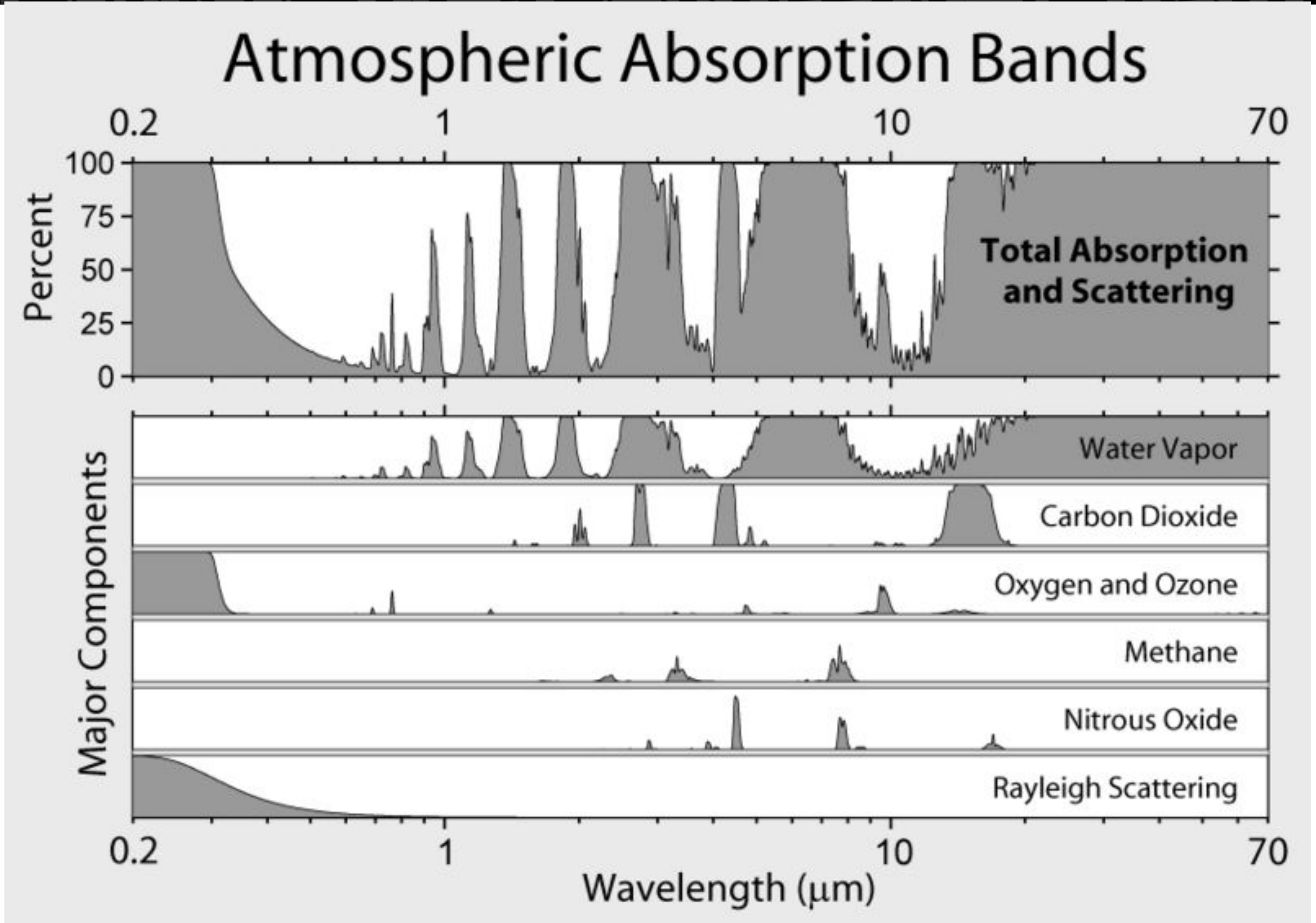
- Many conditions require consideration of both effects

  Together ➡ Voigt profile!

Source:
https://cefrc.princeton.edu/sites/cefrc/files/Files/2013%20Lecture%20Notes/Hanson/pLecture6.pdf

Line by Line Calculation

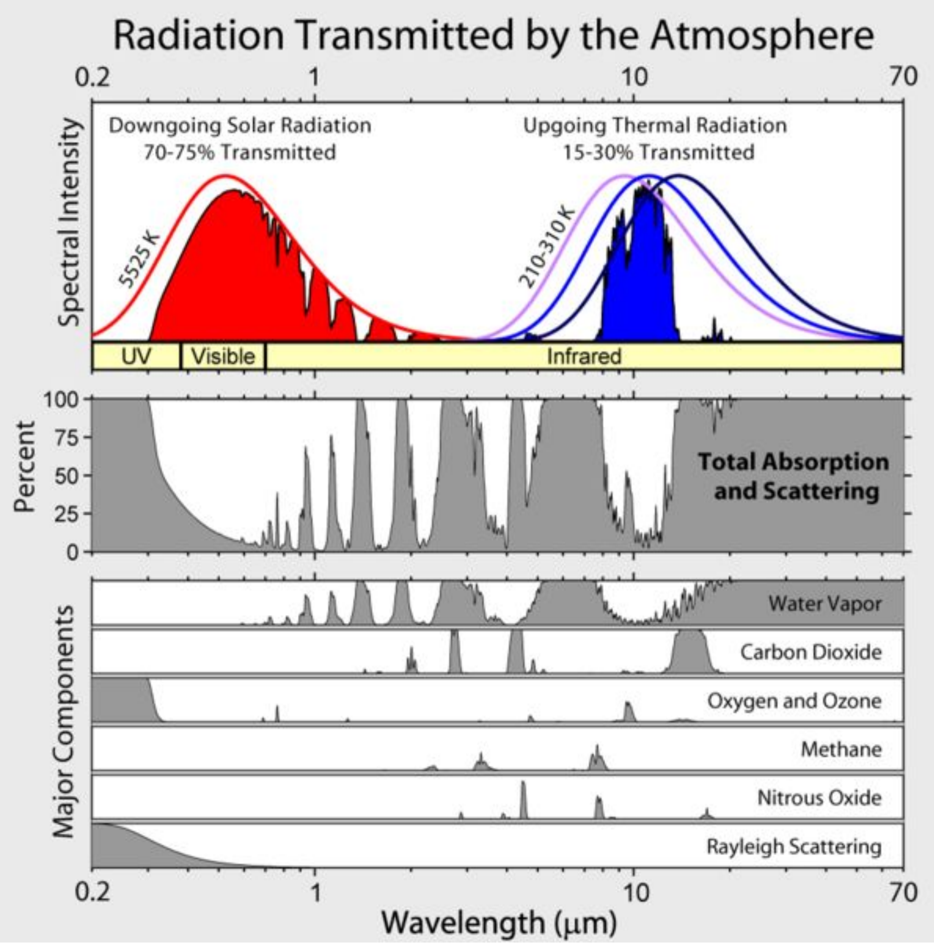



Atmospheric Absorption Bands
0.2
1
10
70
100
75
50
25
0
Percent
Total Absorption and Scattering
Major Components
Water Vapor
Carbon Dioxide
Oxygen and Ozone
Methane
Nitrous Oxide
Rayleigh Scattering
Wavelength (μm)

Gas	Center ν (cm^{-1}) (λ(μm))	Band interval (cm^{-1})
H_2O	3703 (2.7)	2500-4500
	5348 (1.87)	4800-6200
	7246 (1.38)	6400-7600
	9090 (1.1)	8200-9400
	10638 (0.94)	10100-11300
	12195 (0.82)	11700-12700
	13888 (0.72)	13400-14600
	visible	15000-22600
CO_2	2526 (4.3)	2000-2400
	3703 (2.7)	3400-3850
	5000 (2.0)	4700-5200
	6250 (1.6)	6100-6450
	7143 (1.4)	6850-7000
O_3	2110 (4.74)	2000-2300
	3030 (3.3)	3000-3100
	visible	10600-22600
O_2	6329 (1.58)	6300-6350
	7874 (1.27)	7700-8050
	9433 (1.06)	9350-9400
	13158 (0.76)	12850-13200
	14493 (0.69)	14300-14600
	15873 (0.63)	14750-15900
N_2O	2222 (4.5)	2100-2300
	2463 (4.06)	2100-2800
	3484 (2.87)	3300-3500
CH_4	3030 (3.3)	2500-3200
	4420 (2.20)	4000-4600
	6005 (1.66)	5850-6100
CO	2141 (4.67)	2000-2300
	4273 (2.34)	4150-4350
NO_2	visible	14400-50000

Main Visible and near-IR absorption bands of atmospheric gases

Atmospheric gas absorption



Beer-Lambert Law (Absorption and Extinction)

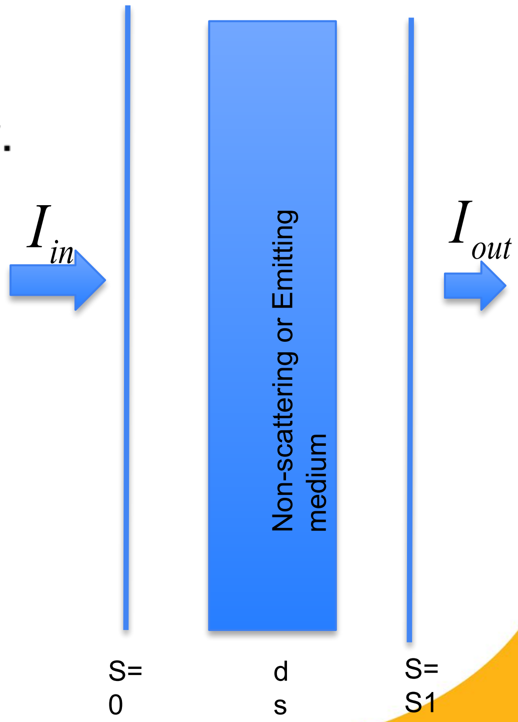
$$-\frac{dI}{\beta_a ds} = I$$

β_a [m^{-1}]: absorption coefficient
ds [m] : path increment
$\beta_a = \rho k_a$: k_a [m^2 kg^{-1}]: mass absorption coefficient, ρ is density.

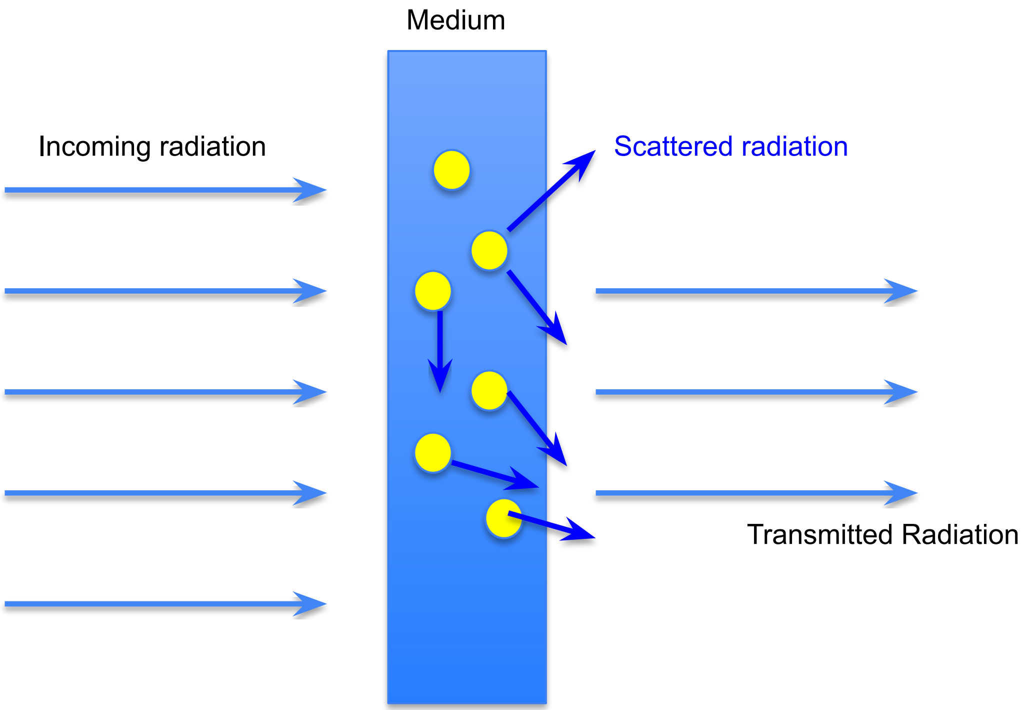
$$I(s) = I(0)\exp\left(-\int_0^s \beta_a ds'\right) \qquad \tau = \int_0^s \beta_a ds'$$

$$I_\lambda(s) = I_\lambda(0)e^{-\tau}$$ Transmitted radiation

$$T_\lambda(s) = I_\lambda(s)/I_\lambda(0) = e^{-\tau}$$ Transmittance



Absorption & Scattering



Energy conservation requires:
Incoming - Transmitted = Scattering + Absorption

Absorption & Scattering

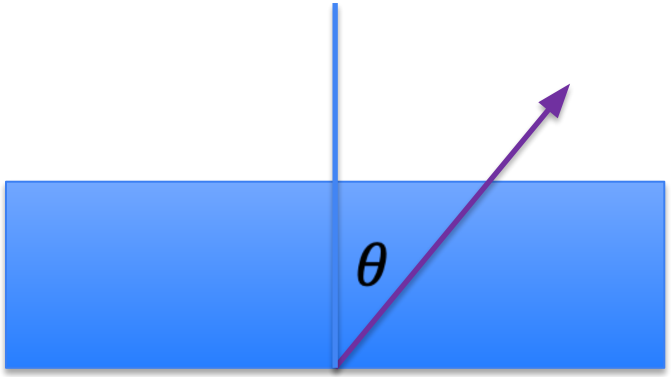
$$\frac{dI}{ds} = -(\beta_a + \beta_s)I + J_s$$

J_s: Source Term due to scattering

$$J_s = \frac{\beta_s}{4\pi} \int_{4\pi} p(\widehat{\Omega}, \widehat{\Omega}')I(\widehat{\Omega}')d\widehat{\Omega}'$$

$$\mu \frac{dI}{d\tau} = -I + \frac{\omega}{4\pi} \int_{4\pi} p(\widehat{\Omega}, \widehat{\Omega}')I(\widehat{\Omega}')d\widehat{\Omega}'$$

$\omega = \frac{\beta_s}{\beta_a + \beta_s}$: Single Scattering Albedo

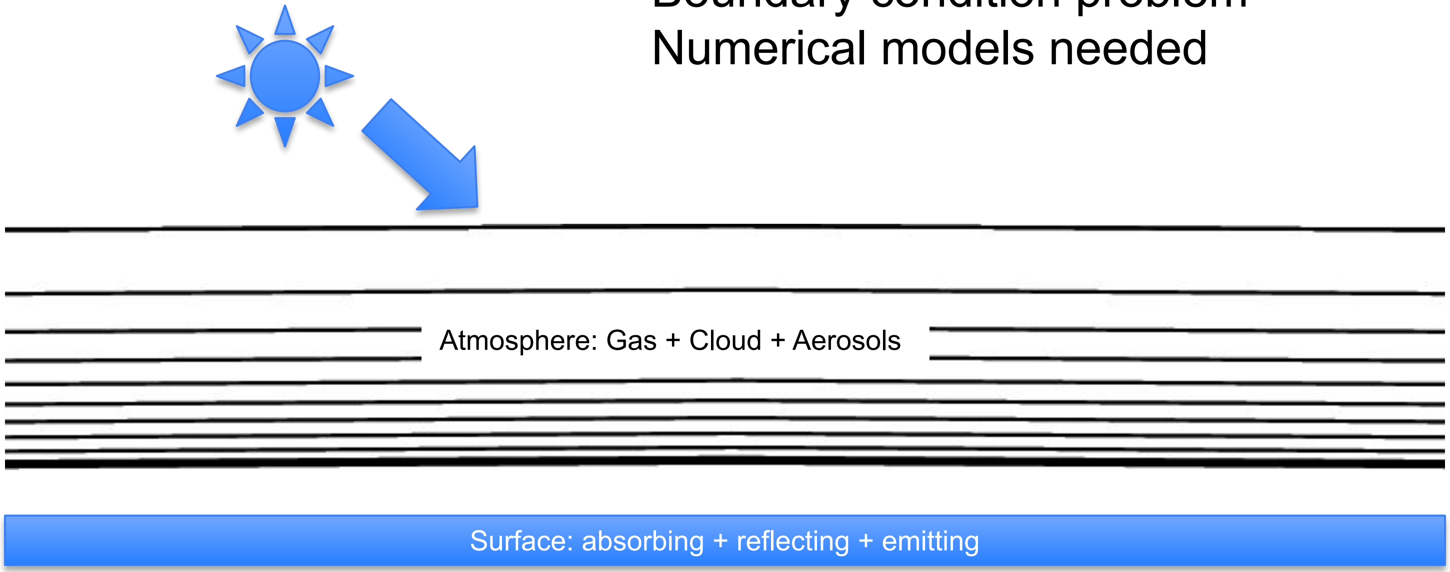


$$\tau = \int_0 (\beta_a + \beta_s)dz'$$

$$ds = \frac{dz'}{\cos\theta} = \frac{dz'}{\mu}$$

Plane-parallel Atmosphere model

Boundary condition problem
Numerical models needed



Radiative Transfer: including emission

$$\frac{dI}{ds} = -(\beta_a+\beta_s)I + J_s + \beta_a \mathrm{B(T)}$$

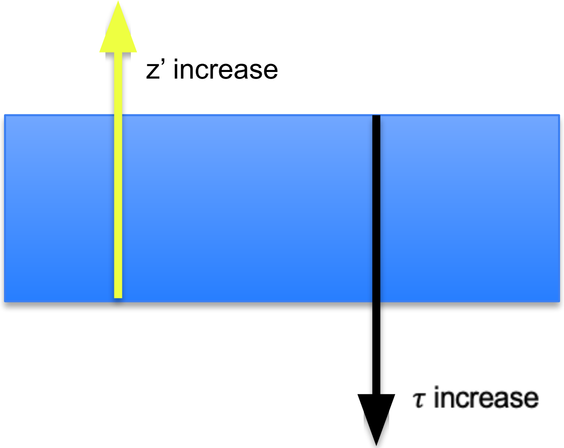
$$\mu\frac{dI}{d\tau} = -I + \frac{\omega}{4\pi}\int_{4\pi} p(\hat{\Omega},\hat{\Omega}')I(\hat{\Omega}')d\hat{\Omega}' + (1-\omega)\mathrm{B(T)}$$

- In solar spectral region (0.2μm~4μm)
  - Emission of Atmosphere is not important (why?)
  - Absorption & Scattering
- In thermal infrared spectral region (>4μm)
  - Pure gas (“clear-sky”): Absorption & Emission
  - Particles (clouds & aerosols) all terms

An alternative form of RTE in a plane parallel medium

Optical thickness due to extinction: $d\tau = -\beta_e dz'$

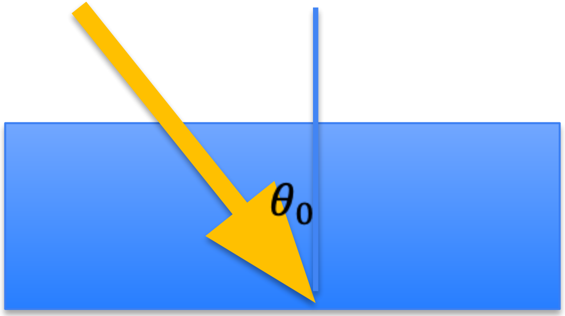
$$\tau = \int_z^\infty \beta_e dz'$$



$$\mu \frac{dI}{d\tau} = -I + \frac{\omega}{4\pi}\int_{4\pi} p(\widehat{\Omega},\widehat{\Omega}')I(\widehat{\Omega}')d\widehat{\Omega}' + (1-\omega)B(T)$$

$$\mu \frac{dI}{d\tau} = I - \frac{\omega}{4\pi}\int_{4\pi} p(\widehat{\Omega},\widehat{\Omega}')I(\widehat{\Omega}')d\widehat{\Omega}' - (1-\omega)B(T)$$

Solar Source to RT



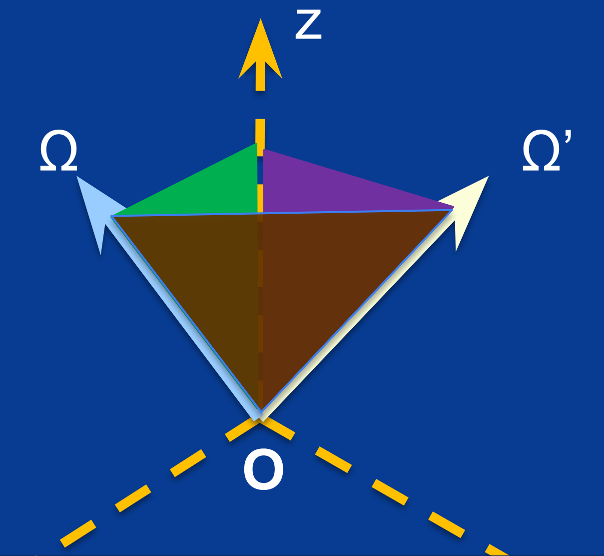
$$\mu \frac{dI}{d\tau} = I - \frac{\omega}{4\pi} \int_{4\pi} p(\widehat{\Omega}, \widehat{\Omega}') I(\widehat{\Omega}') d\widehat{\Omega}'$$

$$I(\widehat{\Omega}') = F_0 e^{-\tau/|\mu_0|} \delta(\mu' - \mu_0) \delta(\phi' - \phi_0) + I_d(\mu', \phi')$$

$$\mu \frac{dI}{d\tau} = I - \frac{\omega}{4\pi} \int_{4\pi} p(\widehat{\Omega}, \widehat{\Omega}') I(\widehat{\Omega}') d\widehat{\Omega}' - \frac{\omega}{4\pi} F_0 e^{-\frac{\tau}{|\mu_0|}} p(\mu, \phi, \mu_0, \phi_0)$$

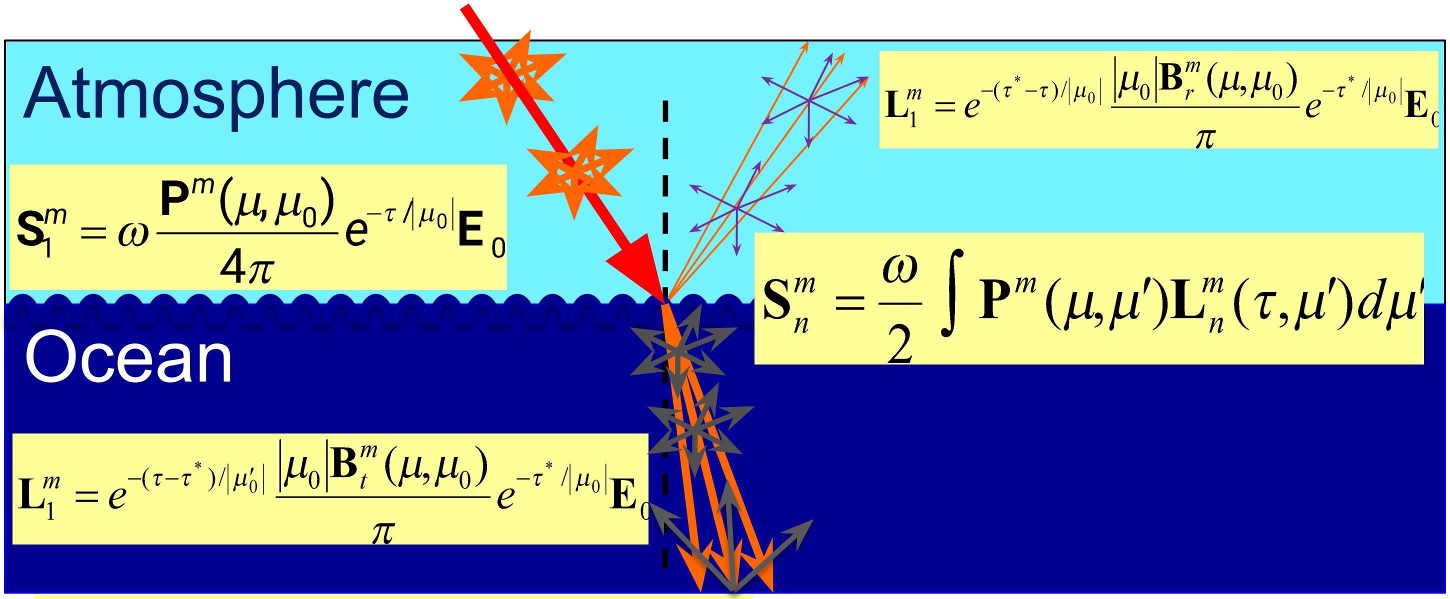
Here we have redefined I as I_d, the diffuse radiance in a turbid medium.

Tracing the Reference Plane



$$Z = R(\pi - i_2)P(\Omega' \cdot \Omega)R(-i_1)$$

RT Solution: Successive Order of Scattering Method



$$\mathbf{S}_1^m = \omega \frac{\mathbf{P}^m(\mu,\mu_0)}{4\pi} e^{-\tau/|\mu_0|}\mathbf{E}_0$$

$$\mathbf{L}_1^m = e^{-(\tau^*-\tau)/|\mu_0|} \frac{|\mu_0|\mathbf{B}_r^m(\mu,\mu_0)}{\pi} e^{-\tau^*/|\mu_0|}\mathbf{E}_0$$

$$\mathbf{S}_n^m = \frac{\omega}{2}\int \mathbf{P}^m(\mu,\mu')\mathbf{L}_n^m(\tau,\mu')d\mu'$$

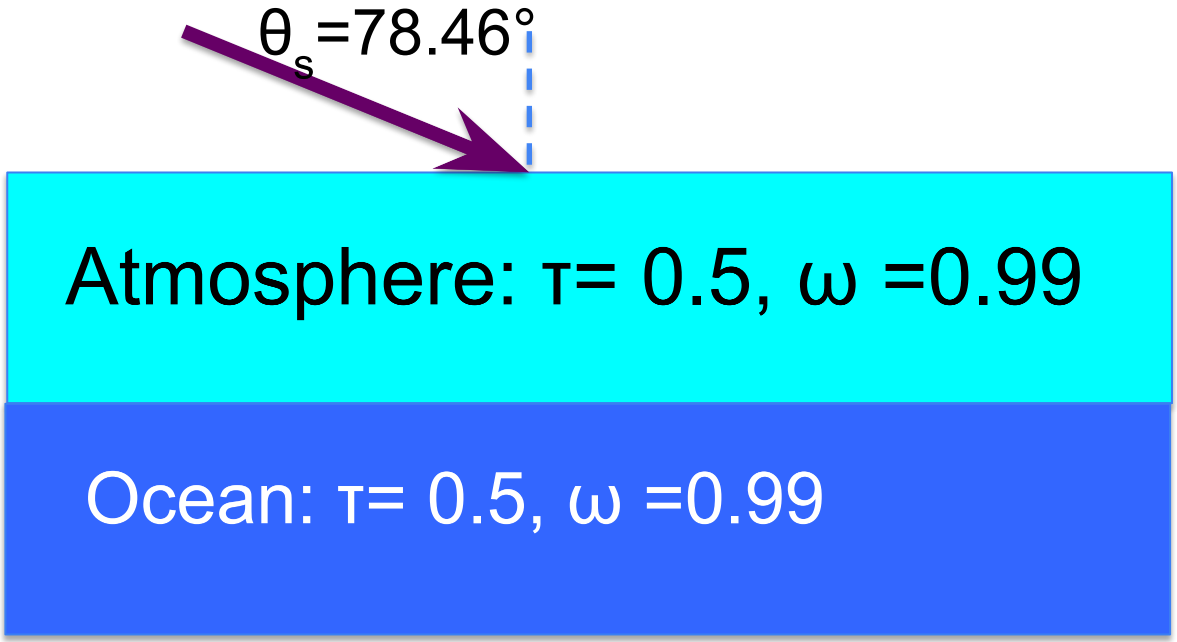
$$\mathbf{L}_1^m = e^{-(\tau-\tau^*)/|\mu_0'|} \frac{|\mu_0|\mathbf{B}_t^m(\mu,\mu_0)}{\pi} e^{-\tau^*/|\mu_0|}\mathbf{E}_0$$

$$\mathbf{L}_n^m(\tau,\mu>0) = \int_{\tau}^{\tau^*} \exp\{-(\tau'-\tau)/\mu\}\mathbf{S}_n^m(\tau',\mu)d\tau/\mu$$

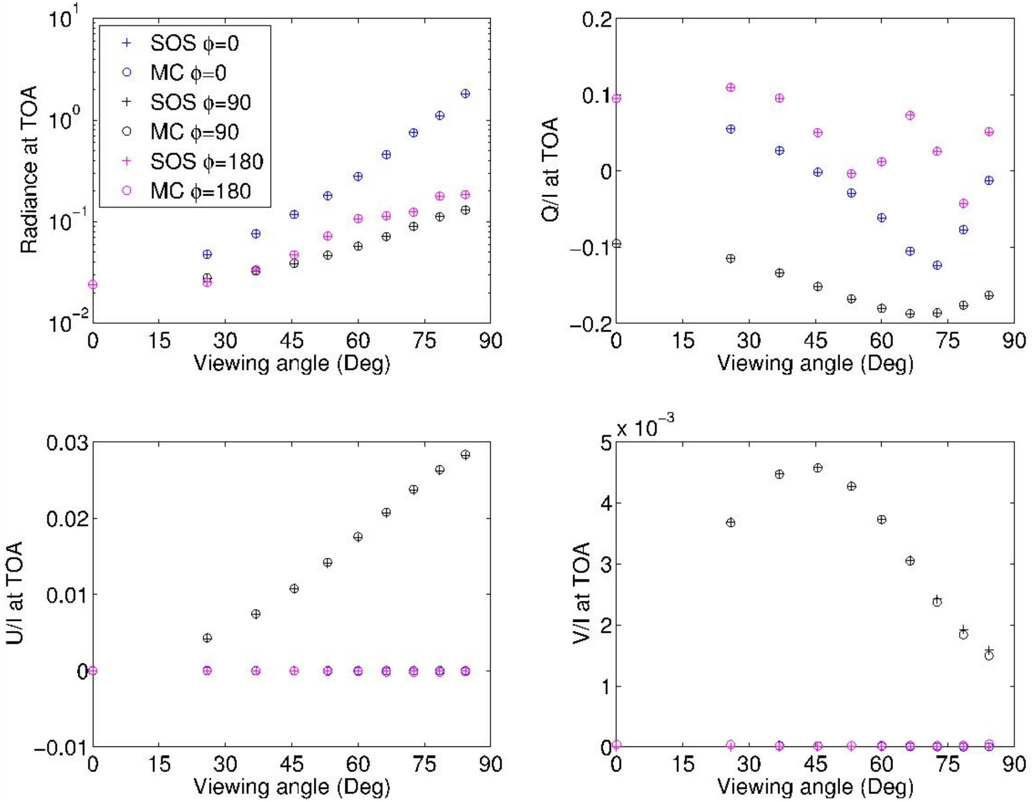
$$\mathbf{L}_n^m(\tau,\mu<0) = \int_{0}^{\tau} \exp\{-(\tau'-\tau)/\mu\}\mathbf{S}_n^m(\tau',\mu)d\tau/|\mu|$$

Zhai, P, Y. Hu, J. Chowdhary, C. R. Trepte, P. L. Lucker, D. B. Josset, “A vector radiative transfer model for coupled atmosphere and ocean systems with a rough interface,” J Quant Spectrosc Radiat Transf, **111**, 1025-1040 (2010).
Zhai, P, Y. Hu, C. R. Trepte, and P. L. Lucker, "A vector radiative transfer model for coupled atmosphere and ocean systems based on successive order of scattering method," Opt. Express **17**, 2057-2079 (2009).

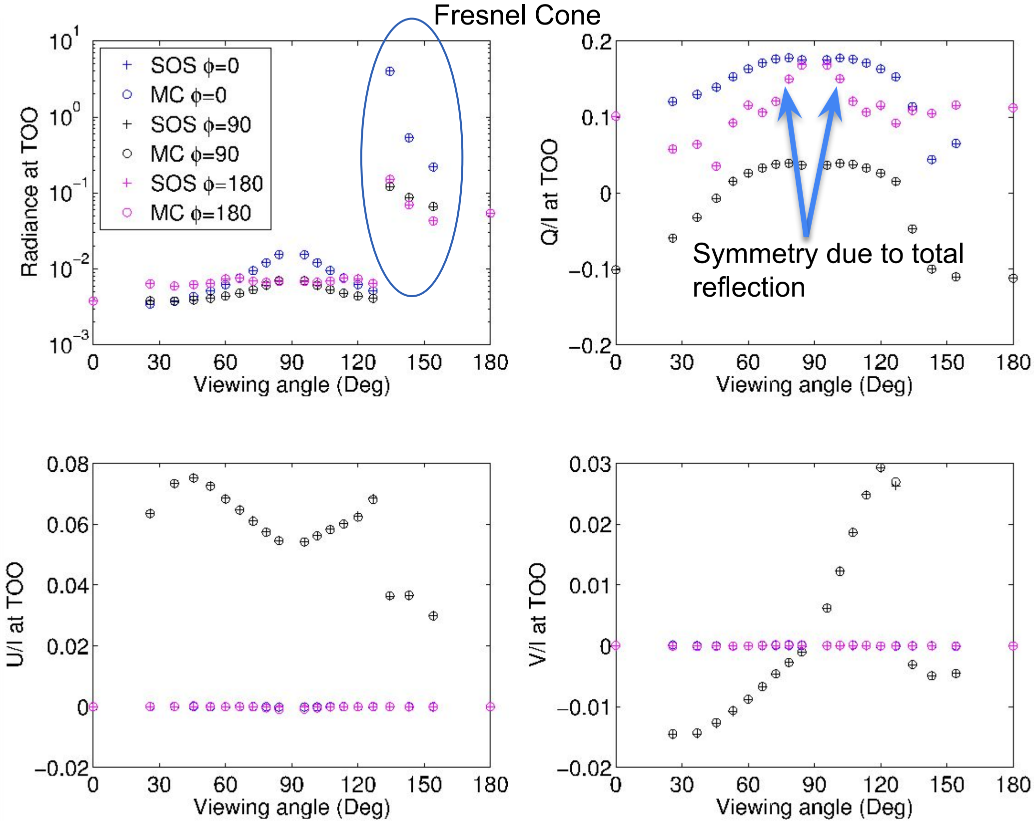
Example Case Configuration



Polarized Radiance Field at TOA



Polarized Radiance Field at TOO



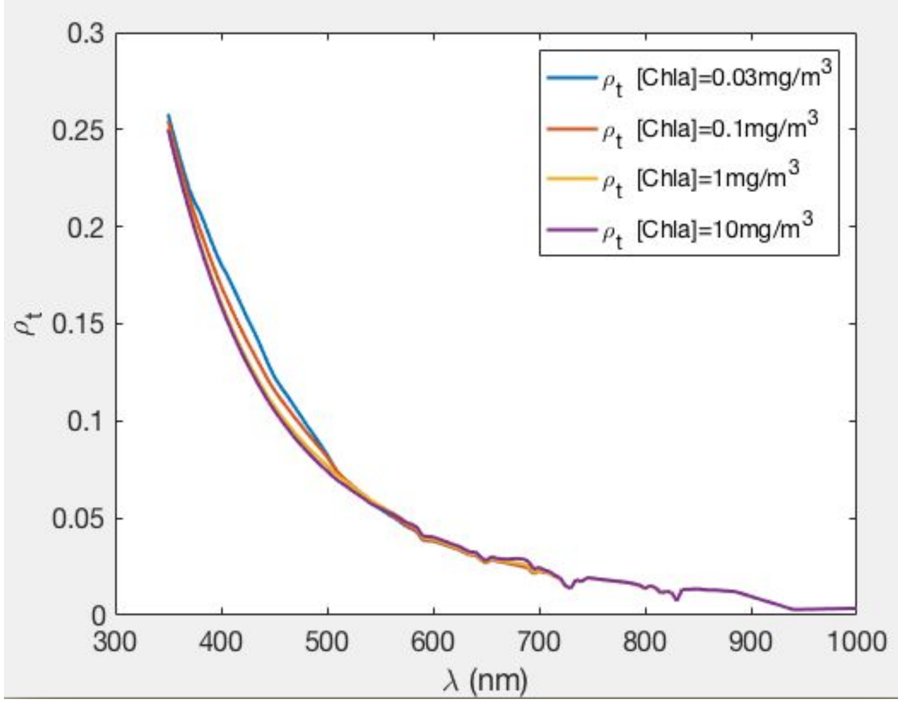
Radiative transfer solutions (A non-complete list)

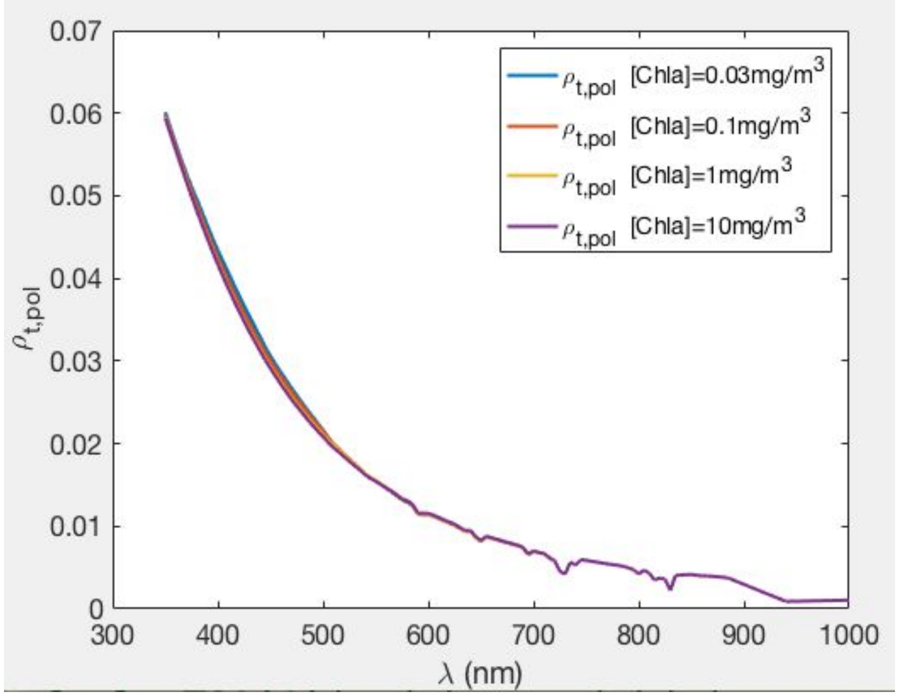
- Discrete Ordinates Radiative Transfer (DISORT) (Liou 1973, Stamnes et al. 1988; Thomas and Stamnes, 1999)
- VDISORT (Weng et al. 1992)
- Coupled Ocean Atmosphere Radiative Transfer (COART) (Jin and Stamnes 1994, Jin et al. 2006)
- Spherical Harmonic Discrete Ordinate Method (SHDOM, SHDOMPP) (Evans 1998, 2006)
- Matrix Operator Method (Plass et al. 1973, Kattawar et al. 1973, Nakajima and Tanaka 1983, Liu 1996, He 2005, Fell and Fischer 2001).
- Adding and Doubling (Stokes 1862, Hansen 1971, van de Hulst 1980, Chowdharry 1999)
- Multicomponent Approach (Zege et al. 1993).
- Invariance and imbedding (Ambartzumian 1942, Chandrasekhar 1960, Adams and Kattawar 1970, Mishchenko 1997)
- The Succesive Order of Scattering (SOS) method (Chami 2001, Min 2004, Lenoble et al. 2007, Zhai et al. 2009)
- Monte Carlo method (Plass and Kattawar 1968, Kattawar and Plass 1968, Marchuk et al. 1980, Roberti and Kummerow, 1999, O’Brien 1992.)

Roles of Radiative Transfer in Remote Sensing Theory

- Explore the physical processes of light transport/propagation in turbid media.
- Perform sensitivity Study for different microphysical parameters.
- Build Look-Up Tables for aerosol or cloud reflectance.
- Act as a forward model for non-linear least squares fitting algorithms.
- Test operational retrieval algorithm.

Application I: TOA Reflectance



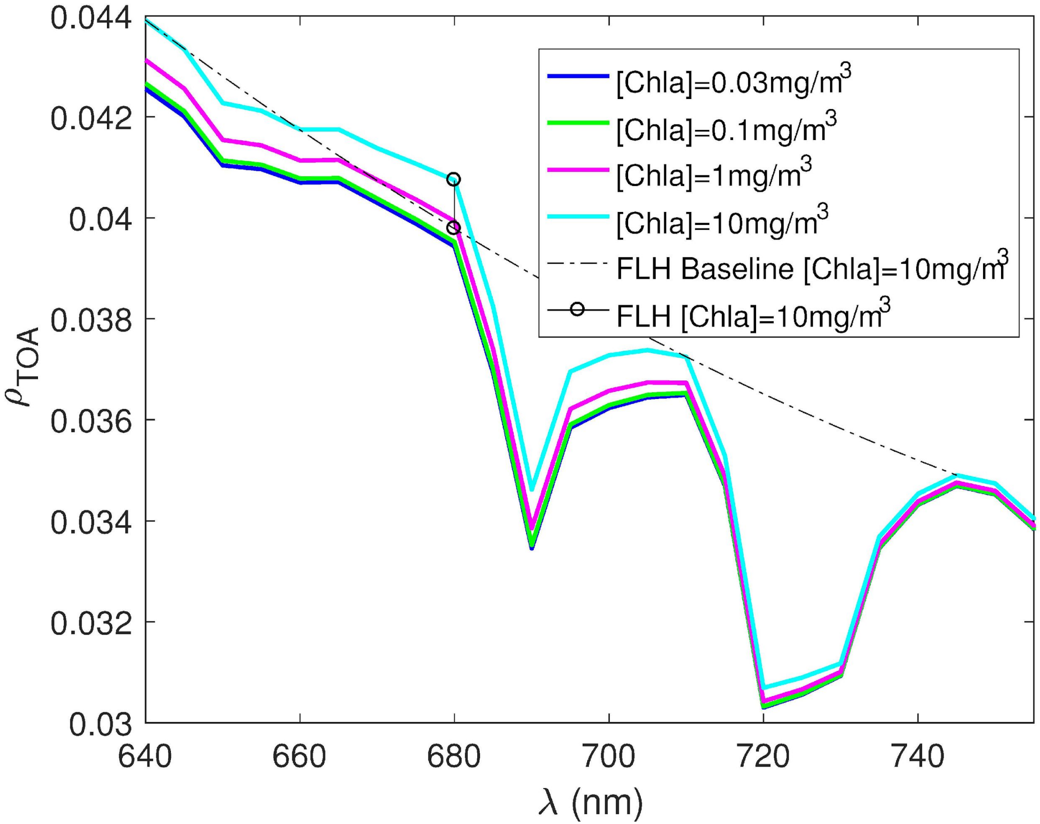


Atmosphere:
$\tau_{a,550\ nm}$=0.1
Aerosol model: Ahmad et al, 2010
Fine-mode volume fraction: 20%
Relative humidity: 80%
Gas absorption: H_2O.

Ocean water model:
pure sea water,
phytoplankton covariant particle, CDOM

Solar zenith angle: 45 degree.

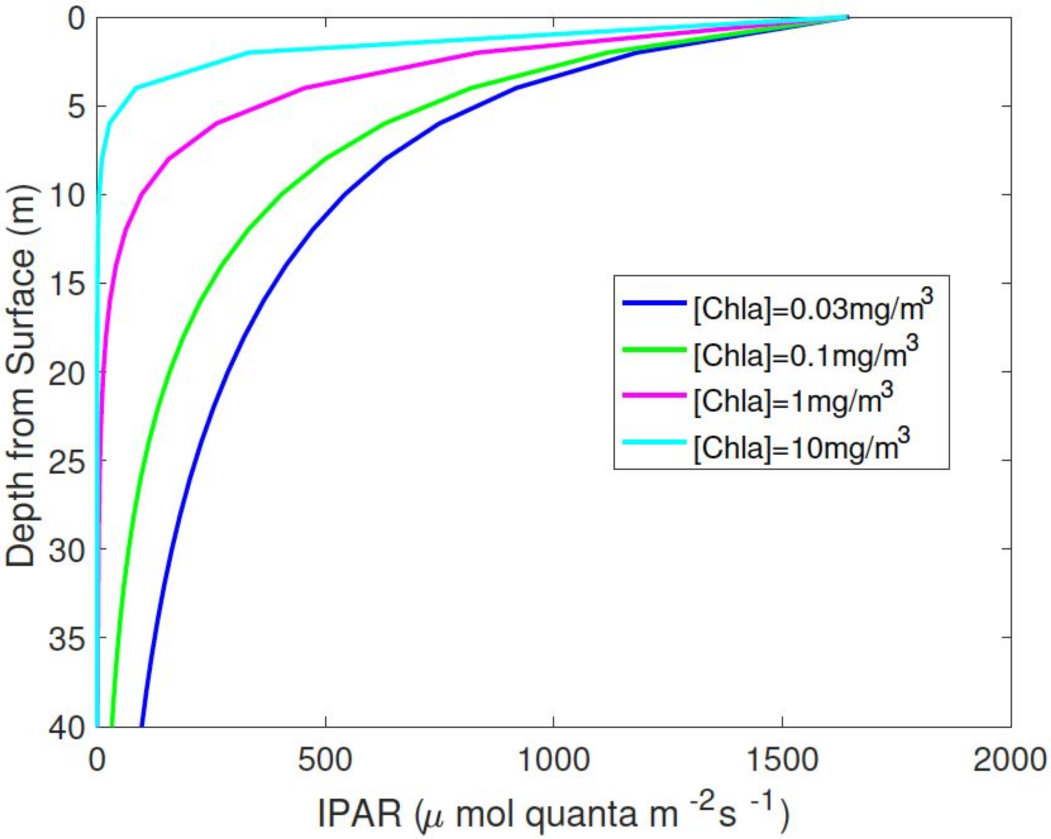
Application II: Chlorophyll a Fluorescence …



Solar zenith angle: 30 degree.
$\tau_{a,550\ nm}$=0.1, Aerosol model: Marinetime
Gas absorption: (H_2O, CO_2, CH_4, O_2, O_3, NO_2)

Ocean water model:
pure sea water,
phytoplankton covariant particle, CDOM

Application III: Modeling IPAR



Reference: Zhai, P.-W.; Boss, E.; Franz, B.; Werdell, P.J.; Hu, Y. Radiative Transfer Modeling of Phytoplankton Fluorescence Quenching Processes. *Remote Sens.* **2018**, *10*, 1309.

Application IV: MAPOL Joint Retrieval Algorithm

Retrieval Optimization

Forward Model (VRT for CAOS, Zhai. et al., (2009, 2011)**)**

Aerosol Model
- Six sub mode volume distribution (3 fine modes, 3 coarse modes)
- Refractive index based on PCA for real and imaginary spectra

Bio-optical Models
- Open waters – case 1 (Phytoplankton)
- Coastal waters – case 2 (Phytoplankton, CDOM, NAP)

Rough ocean interface
(Isotropic Cox-Munk Model)

Levenburg- Marquardt non-linear least squares optimization (More et al., 1980)

Cost function (χ^2)

$$\chi^2(x) = \frac{1}{N}\sum_i \left(\frac{\left[\rho_t^m(i) - \rho_t^f(x,i)\right]^2}{\sigma_t^2(i)} + \frac{\left[A^m(i) - A^f(x,i)\right]^2}{\sigma_A^2(i)}\right)$$

RSP: A – Polarized Reflectance
SPEX: A – DOLP

Parameters

Aerosol refractive index spectra – 8
Aerosol volume distribution – 6
Wind speed – 1
Hydrosol optical properties
 [Chla] based model – 1
 General model – 7

Gao M. et al., (2018,2019)

Take home messages

- Light interacts with media thorough scattering, absorption and extinction, scattering cross section + absorption cross section=extinction cross section.
- The main scattering particles in the Earth systems are molecules, aerosols, clouds, and hydrosols.
- Light scattering solves Maxwell's equation in the frequency domain subjects to matching conditions, which is mainly dominated by refractive index, particle shape, and size.
- Light scattering provides the single scattering properties (Mueller scattering matrix, cross sections), which are used in multiple scattering theory of light, radiative transfer.
- Radiative transfer governs light multiply scattered in a turbid medium, which is a function of wavelength, viewing geometry, and sensor locations.
- Polarization is a powerful tool for characterizing the scattering medium, which can be used to greatly reduce the remote sensing uncertainty.